Anelastic vertical vorticity equation

Taking $\hat{k} \cdot (\nabla \times$ the anelastic equations of motion yields the anelasic vertical vorticity equation:

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{\partial u}{\partial z}\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\frac{\partial w}{\partial x} - \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

No baroclinic term in here (no p or ρ). Baroclinicity is very important in convective storms, but the baroclinic vector is mostly horizontal.

This equation relates w to horizontal wind components u, v. **Can be used as a constraint in dual-Doppler wind analysis.**

3DVAR analysis with vorticity equation constraint

Seek u, v, w that minimize the sum of errors in the analysis constraints:

$$J \equiv \int\int\int\int \left(\alpha_1 O_1^2 + \alpha_2 O_2^2 \right) dr\, d\theta\, d\phi\, dt \quad +$$

$$\int\int\int \left(\delta \varepsilon_m^2 + \gamma \varepsilon_v^2 + \beta_1 S_1 + \beta_2 S_2 + \beta_3 S_3 + \beta_4 S_4 \right) dx\, dy\, dz.$$

O_1, O_2: Differences between analyzed and observed v_r data.

ε_m: Residual in mass conservation equation.

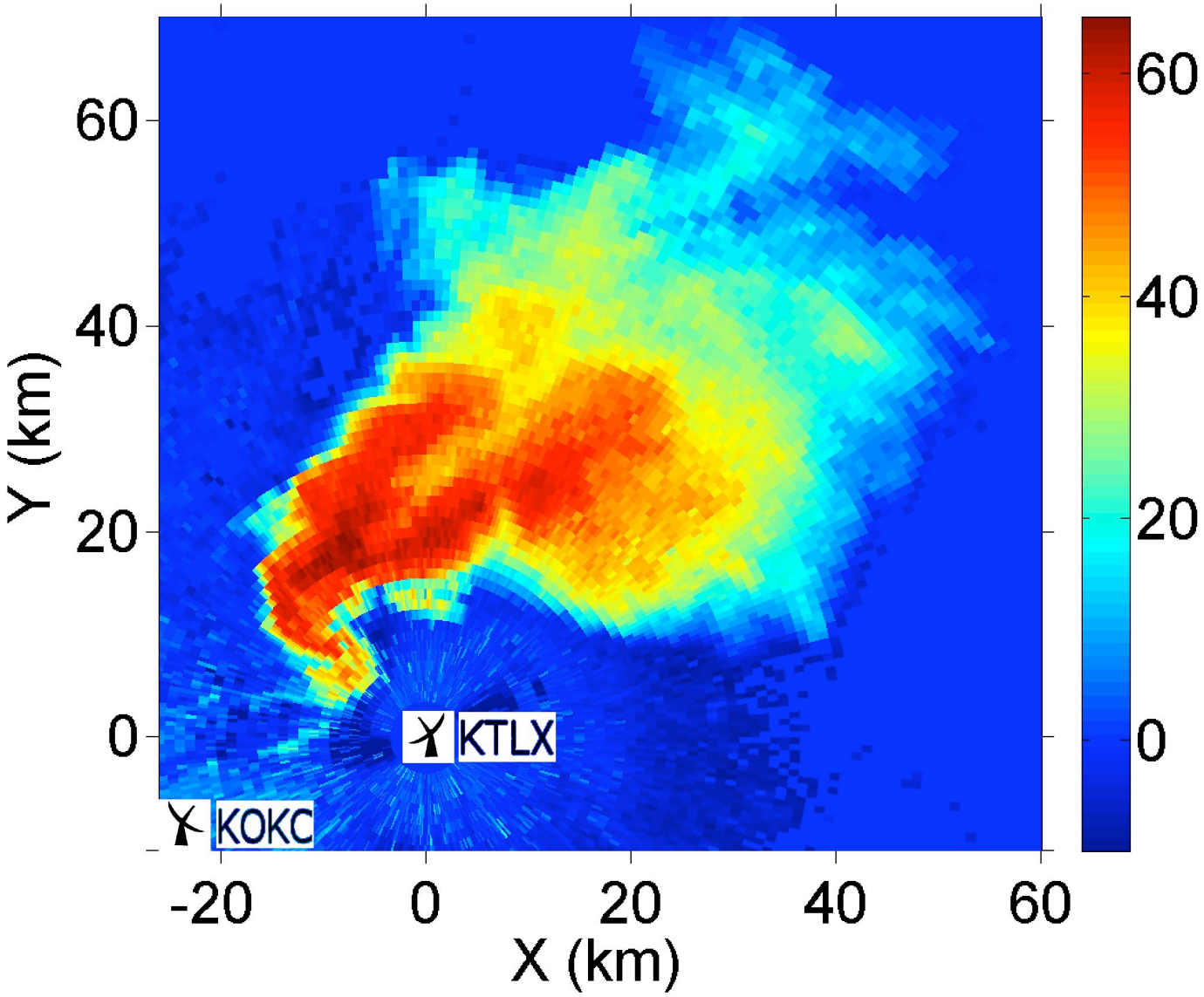
ε_v: Residual in anelastic vertical vorticity equation.

$S_1 - S_4$: Squared spatial derivatives of u, v, w (smoothness terms).

$\beta_1 - \beta_4$: Smoothness weights.

J is minimized with a conjugate-gradient algorithm.

Radar locations



Data denial experiments

Control Run ("truth")

No vorticity equation constraint imposed but all other constraints are turned on.

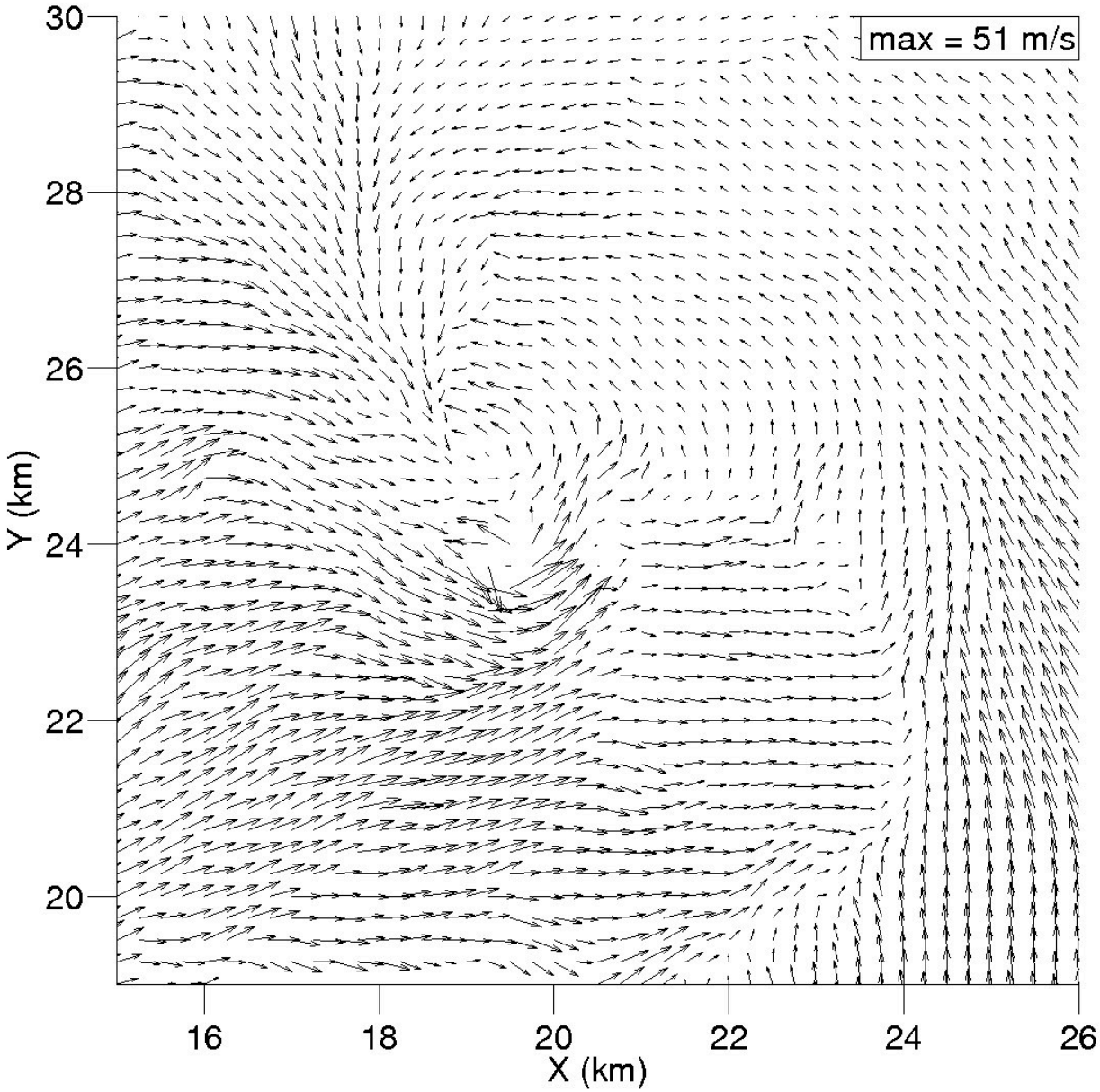
Data Denial Experiment 1: NOVORT

Radial wind data thrown out for $z < 1$ km. Otherwise, experiment is same as control run (no vorticity equation constraint).

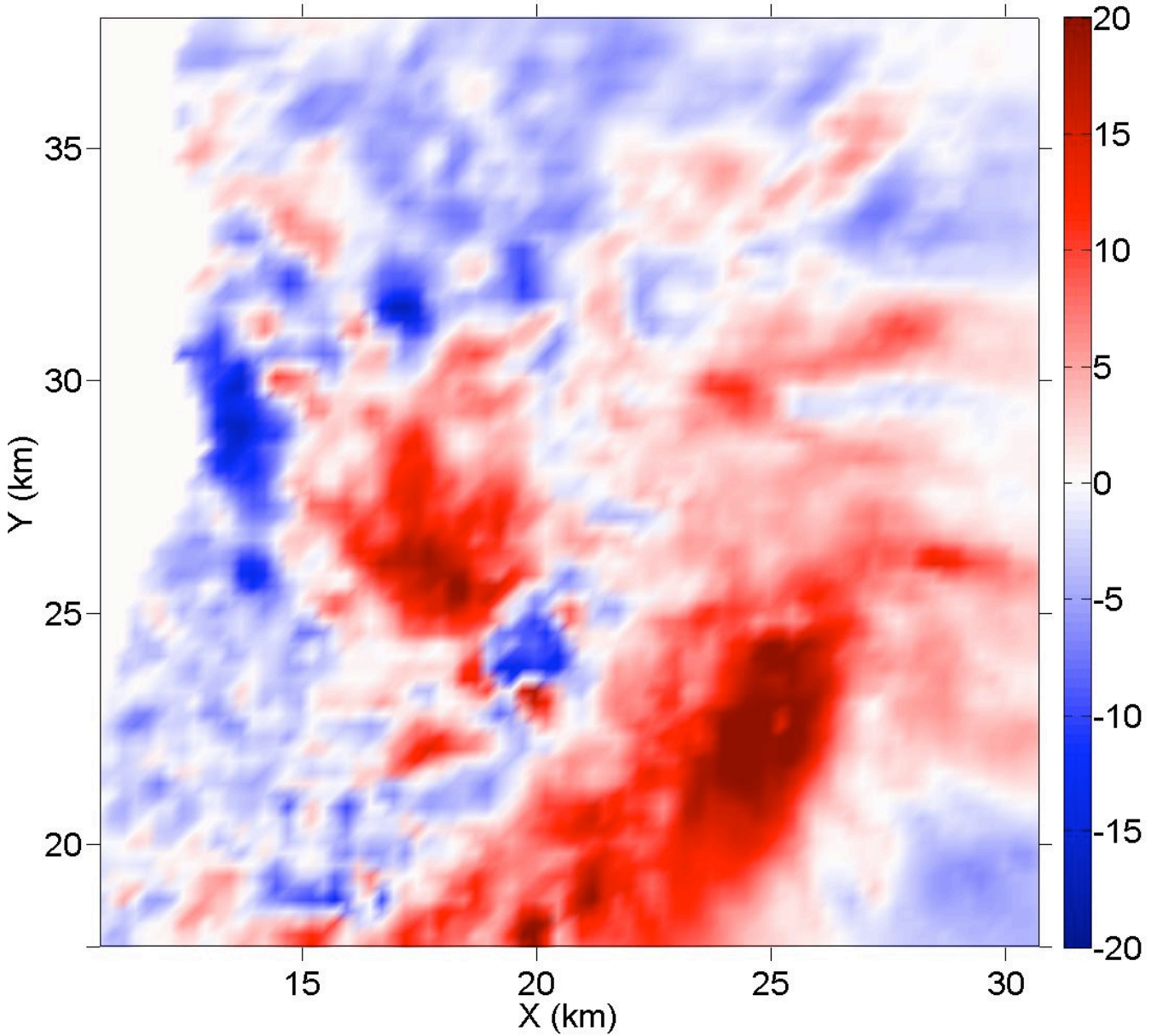
Data Denial Experiment 2: VORT

Radial wind data thrown out for $z < 1$ km. The vorticity equation constraint is turned on.

"True" wind field at z = 0.75 km AGL



"True" *w* (m/s) at *z* = 1.75 km AGL



Impact of vorticity constraint: *w* (m/s) at *z* = 1.75 km AGL