New techniques in dual-Doppler radar wind analysis

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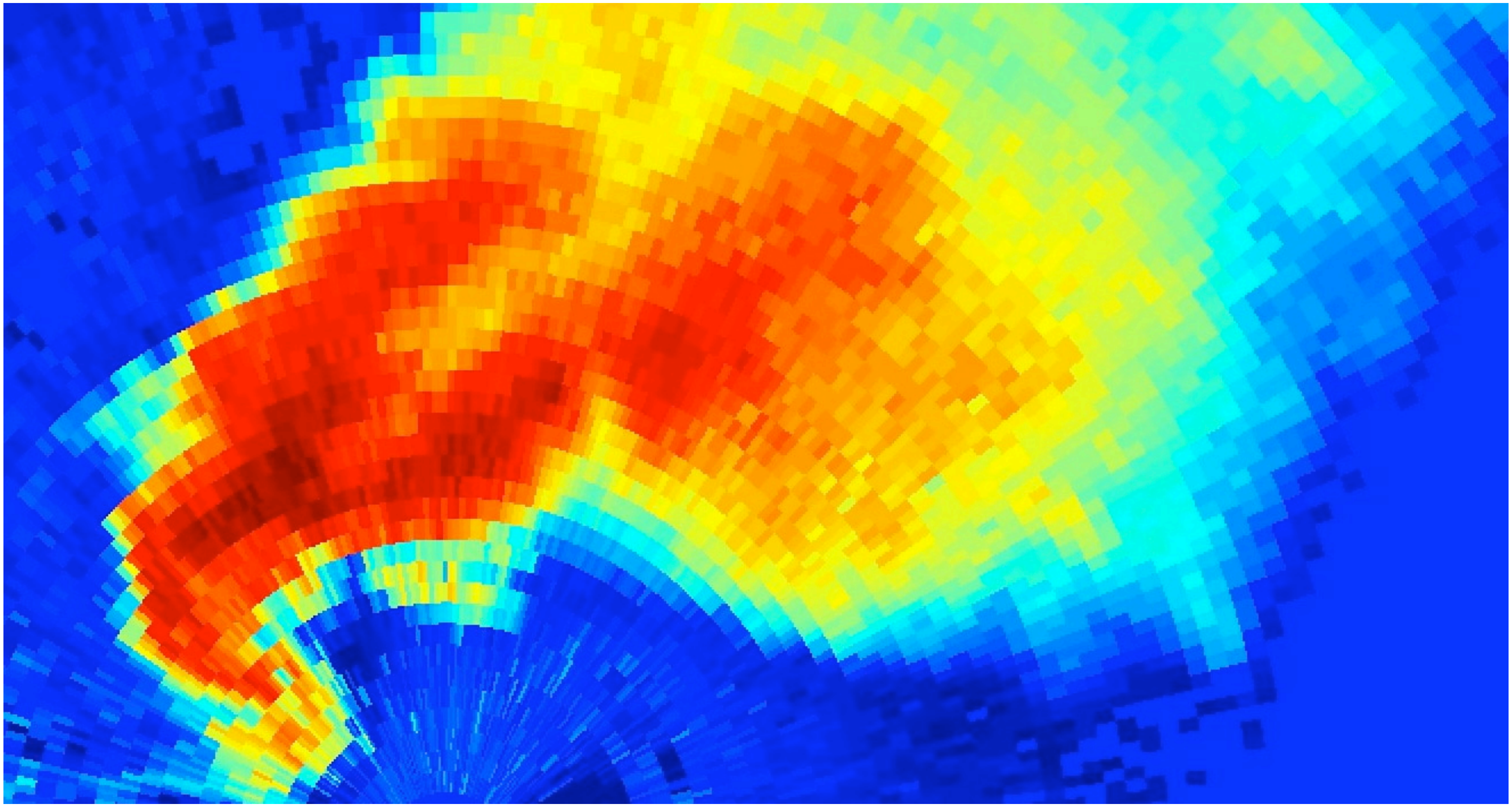
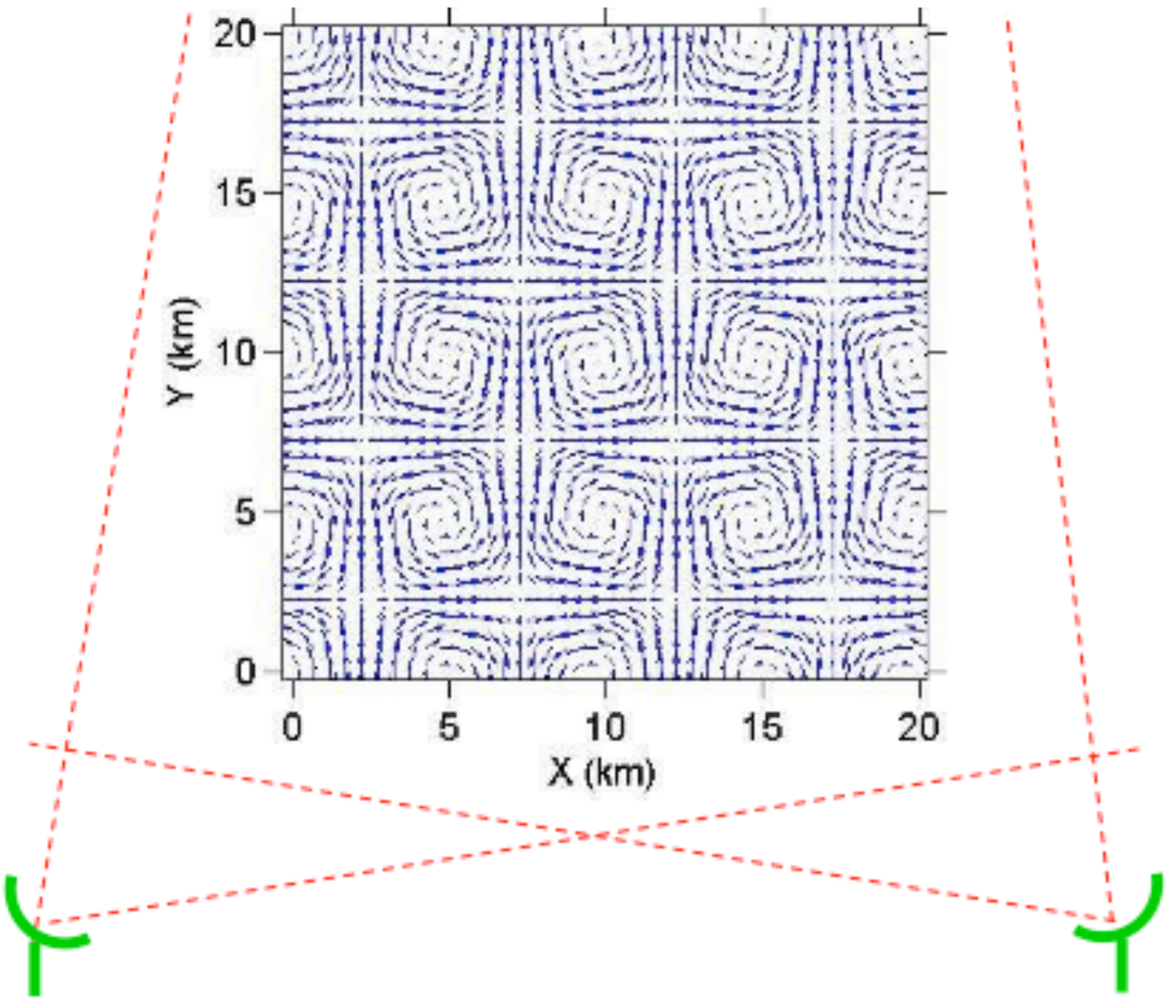


Goal of dual-Doppler wind analysis

Synthesize 3D fields of u, v, w in small- and meso-scale phenomena from volume scans of radial wind data from two Doppler radars.



Main ingredients of dual-Doppler wind analysis

Many different analysis frameworks (Cartesian vs co-plane, direct vs iterative, strong vs weak constraints), but most have same ingredients:

1. Radial wind observations v_r from two radars,
2. Smoothness constraint (explicit or pre/post processing filter),
3. Mass conservation (e.g., $\nabla \cdot [\bar{\rho}(z)\vec{u}] = 0$),
4. Impermeability condition ($w = 0$ at ground level and/or storm top).

An exact theory for dual-Doppler wind analysis

Armijo (1969) derived the solution for a 3D velocity field $\vec{u}$ for which

(i) radial components of $\vec{u}$ agree with radial wind observations,

$$\vec{u} \cdot \hat{r}_1 = v_{r1}, \tag{4}$$

$$\vec{u} \cdot \hat{r}_2 = v_{r2}, \tag{5}$$

(ii) anelastic mass conservation equation is satisfied,

$$\nabla \cdot [\bar{\rho}(z) \vec{u}] = 0, \tag{6}$$

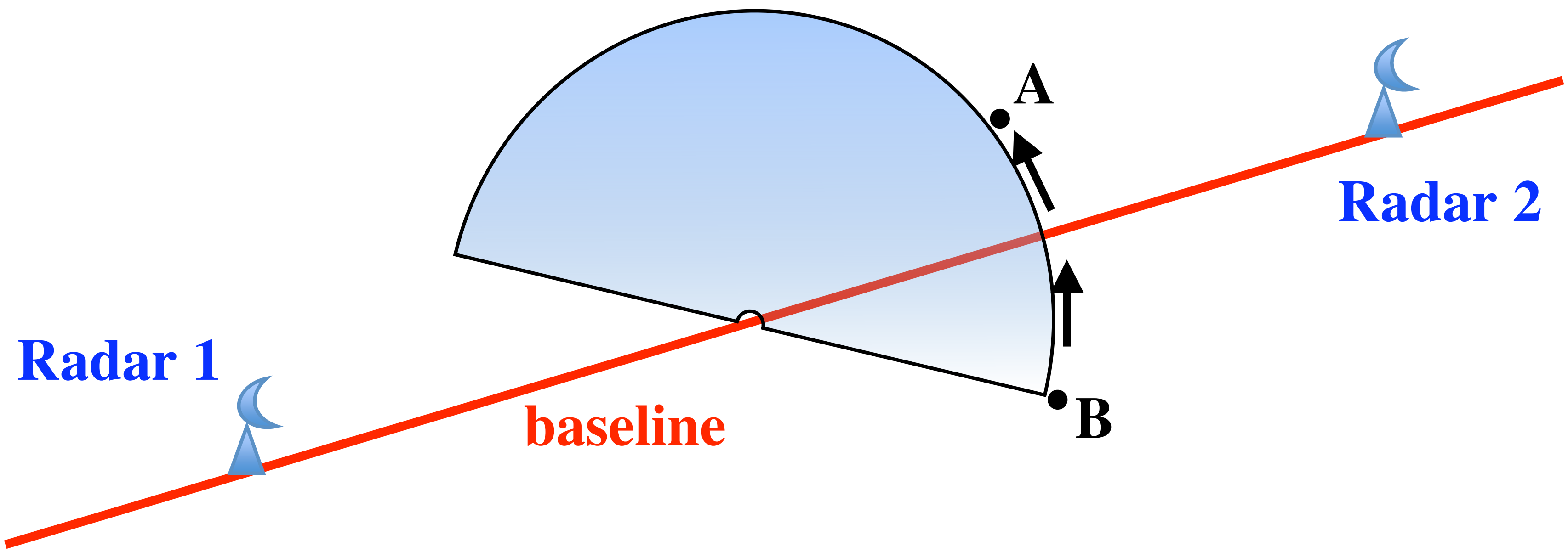
(iii) impermeability condition is satisfied ($w = 0$ at ground level).

The u, v, w fields satisfy (4)–(6). Eliminating u and v in favor of w yields a 1st order partial differential equation for w. Get the exact analytical solution by integrating a forcing term along characteristics.

Coplane coordinate system

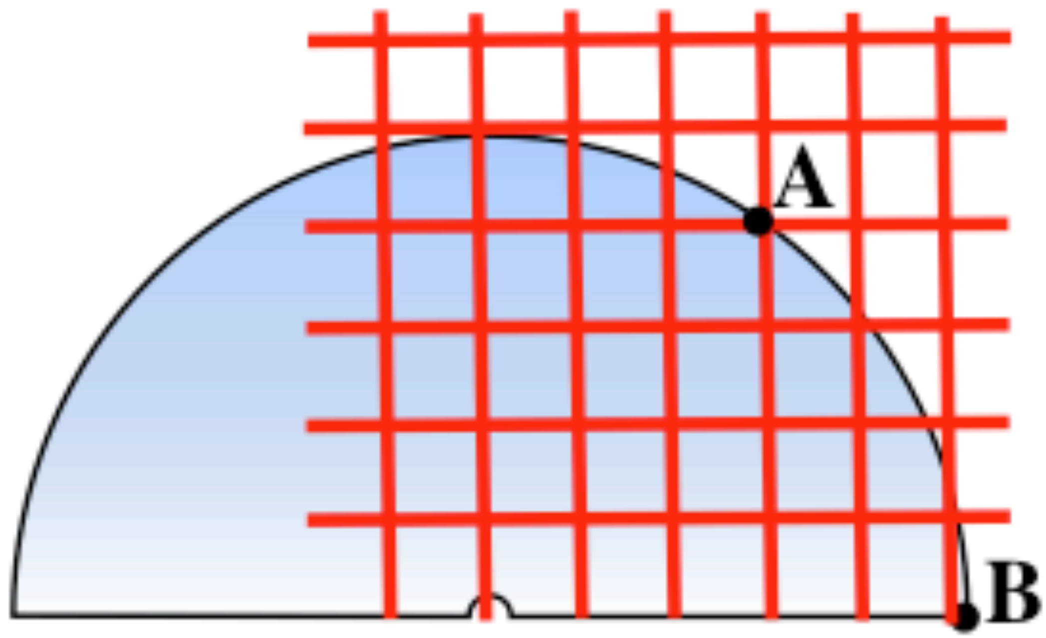
The characteristics in the Armijo theory are circles in a cylindrical coordinate system whose central axis connects the radars (baseline). To get w at any point (e.g., **A**), integrate the forcing term (data) along the circle passing through **A**. Start at the ground (**B)** where $w = 0$.

Well-posedness condition: a unique solution for w exists at **A** if there are data at all points from **A** to **B**. **No solution exists if data are missing between A and B.**



A Cartesian form of dual-Doppler wind analysis

Can bypass Armijo procedure, and solve (4)–(6) iteratively in Cartesian coordinates (e.g., Brandes 1977; Ray et al. 1980; Hildebrand & Mueller 1985; Dowell & Bluestein 1997).



However, the iterative procedure does not always converge. Dowell & Shapiro (2003) derived a stability condition that showed that Armijo's "well-posedness" condition was relevant even in Cartesian coordinates.

Ongoing challenges with dual-Doppler wind analysis: problems and some (partial) solutions

Even in cases where the analysis is well posed (either Armijo's Coplane analysis or the iterative Cartesian analysis), dual-Doppler analyses are still subject to a number of practical difficulties.

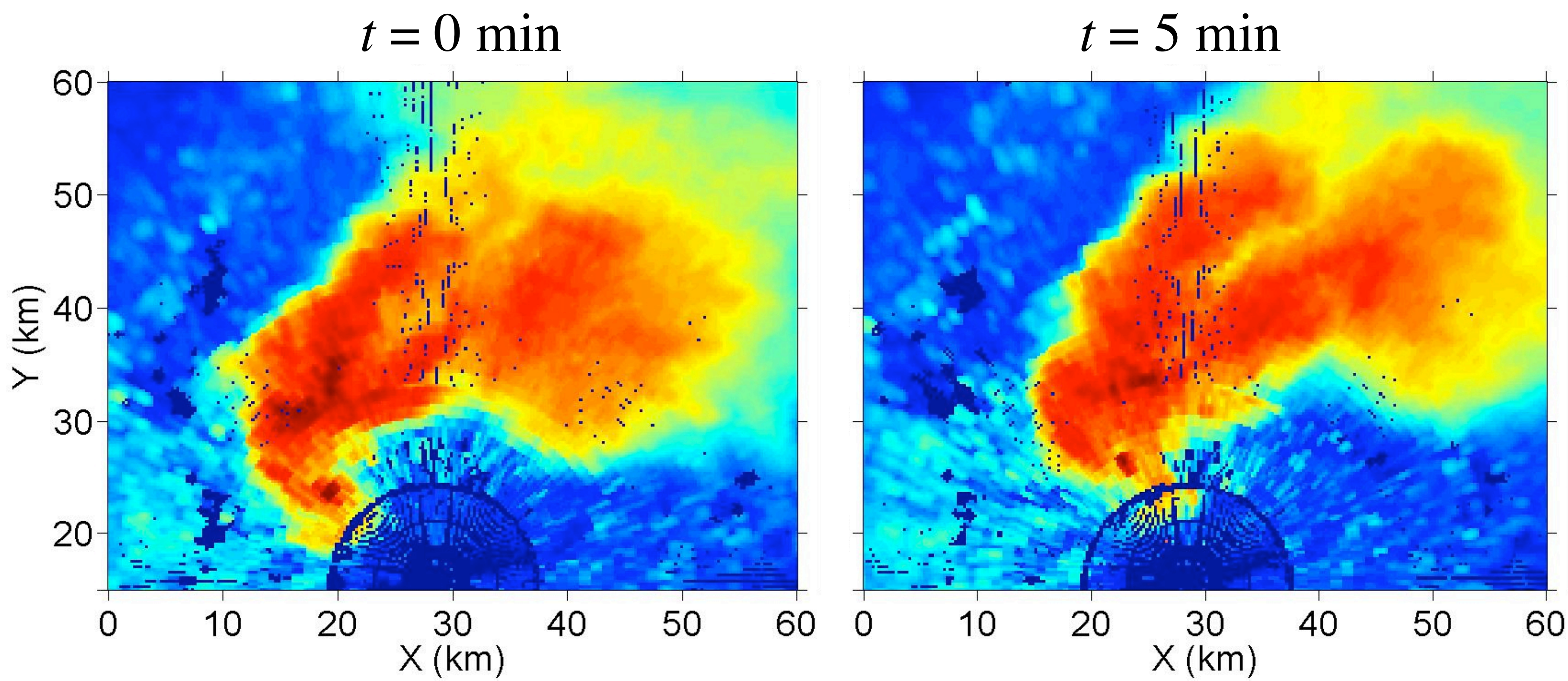
Problem 1: Biases in the divergence can quickly accumulate in the integration process and yield catastrophic errors in *w*.

Solution: Use radial wind data and mass conservation equation $\nabla \cdot [\rho_0(z)\vec{u}] = 0$ as weak constraints (least squares error) in a variational procedure, e.g. 3DVAR or 4DVAR.

We will look at a 3DVAR procedure later.

Problem 2: Non-simultaneous data collection can result in phase (location) errors in key features such as gust fronts and vortices.

Solution: Use "advection correction." Invoke the frozen-turbulence hypothesis to shift data from both radars to a common analysis time.



Frozen-turbulence hypothesis

Frozen-turbulence hypothesis: patterns translate (shift) without change in shape or intensity. In the case of the reflectivity field *Z*, this implies:

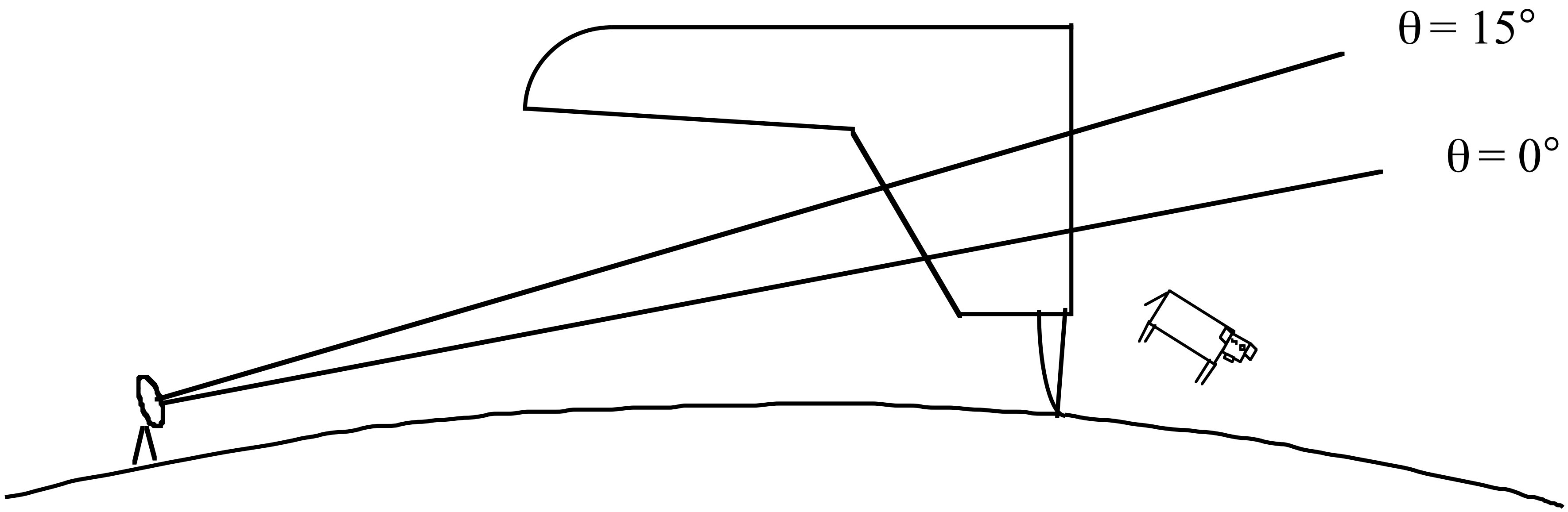
$$\frac{DZ}{Dt} = 0, \qquad \text{or} \qquad \frac{\partial Z}{\partial t} + U\frac{\partial Z}{\partial x} + V\frac{\partial Z}{\partial y} = 0, \tag{7}$$

where *U*, *V* are pattern-translation components (<u>not</u> wind velocity components).

Many methods are available to estimate *U*, *V* (e.g., Gal Chen 1982), however these generally treat *U* and *V* as constants over the whole grid.

We will consider a procedure to derive/use spatially variable *U*, *V* fields in advection correction.

Problem 3: Missing low-level data due to earth curvature, ground clutter, or non-zero elevation angle of lowest sweep.



Solution: Extrapolate data from the lowest sweep down to the ground.

Alternatively, use an additional constraint, e.g. a vorticity equation.

We will also look at this later.

Spatially variable advection correction

We seek $U(x, y)$, $V(x, y)$ and reflectivity $Z(x, y, t)$ fields on horizontal or constant elevation angle surfaces that **minimize the cost function:**

$$J \equiv \iiint \left[\alpha \left(\frac{\partial Z}{\partial t} + U \frac{\partial Z}{\partial x} + V \frac{\partial Z}{\partial y} \right)^2 + \beta \left| \nabla_h U \right|^2 + \beta \left| \nabla_h V \right|^2 \right] dx\, dy\, dt \,, \quad (8)$$

with Z imposed at two effective data times, $t = 0$ and $t = T$.

β is a smoothness coefficient; α is a data coverage function (= 0 or 1) that satisfies $\frac{\partial \alpha}{\partial t} + U \frac{\partial \alpha}{\partial x} + V \frac{\partial \alpha}{\partial y} = 0$.

A similar J underpins some single-Doppler velocity retrievals (Laroche & Zawadzki 1995; Liou & Luo 2001) and some precipitation nowcasting algorithms (Germann & Zawadzki 2002).

Euler-Lagrange equations

Two elliptic equations,

$$\beta T \frac{\partial^2 U}{\partial x^2} + \beta T \frac{\partial^2 U}{\partial y^2} = \int \alpha \frac{\partial Z}{\partial t} \frac{\partial Z}{\partial x} dt + U \int \alpha \left(\frac{\partial Z}{\partial x} \right)^2 dt + V \int \alpha \frac{\partial Z}{\partial x} \frac{\partial Z}{\partial y} dt, \quad (9)$$

$$\beta T \frac{\partial^2 V}{\partial x^2} + \beta T \frac{\partial^2 V}{\partial y^2} = \int \alpha \frac{\partial Z}{\partial t} \frac{\partial Z}{\partial y} dt + U \int \alpha \frac{\partial Z}{\partial x} \frac{\partial Z}{\partial y} dt + V \int \alpha \left(\frac{\partial Z}{\partial y} \right)^2 dt, \quad (10)$$

and one parabolic equation,

$$\alpha \left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} + V \frac{\partial}{\partial y} \right)^2 Z + \alpha \left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} \right) \left(\frac{\partial Z}{\partial t} + U \frac{\partial Z}{\partial x} + V \frac{\partial Z}{\partial y} \right)$$
$$+ \left(\frac{\partial \alpha}{\partial t} + U \frac{\partial \alpha}{\partial x} + V \frac{\partial \alpha}{\partial y} \right) \left(\frac{\partial Z}{\partial t} + U \frac{\partial Z}{\partial x} + V \frac{\partial Z}{\partial y} \right) = 0. \quad (11)$$

Analytical solution of the equation for *Z*

The characteristics of (11) are solutions of the trajectory equations: $Dx/Dt = U$, $Dy/Dt = V$. In characteristic coordinates, (11) becomes:

$$\frac{D^2R}{Dt^2} + \left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right)\frac{DR}{Dt} = 0. \qquad (12)$$

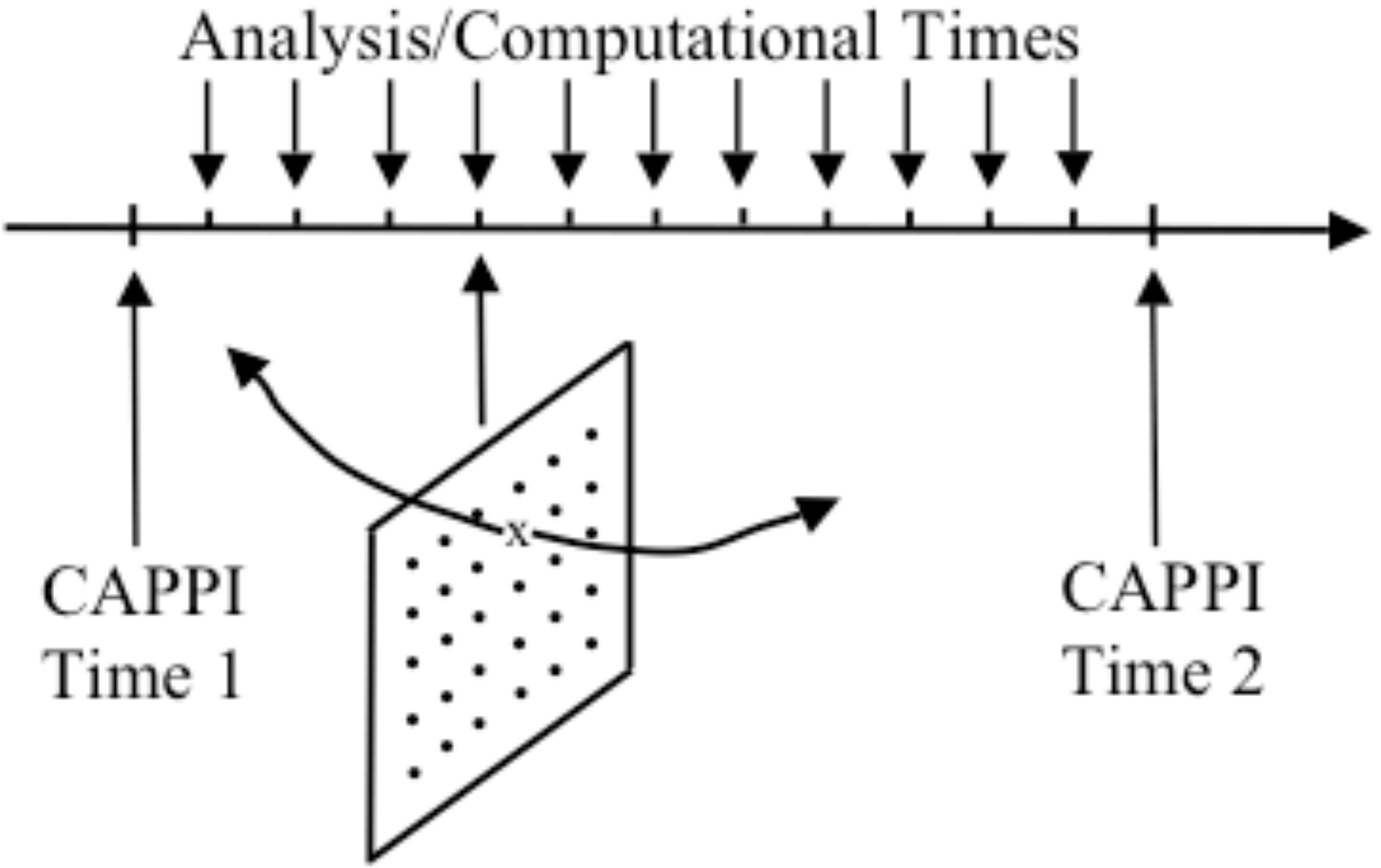
Integrate (13) twice with respect to time along trajectories. Evaluate constants of integration using data at the two input times. Solution is:

$$R(t) = R(t_1) + \left[R(t_2) - R(t_1)\right]\frac{I(t)}{I(t_2)}, \qquad (13)$$

where

$$I(t) \equiv \int_{t_1}^{t} \exp\left[-\int_{t_1}^{t'}\left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right)(t'')dt''\right]dt'. \qquad (14)$$

Analysis grid



Combined analytical/numerical solution

Step 0: Construct CAPPIs at two data input times (2 volume scans)

Then, iterate between these steps:

Step 1: Solve the elliptic equations for U and V by relaxation.

Step 2: Calculate forward and backward trajectories running through all analysis points at all computational times.

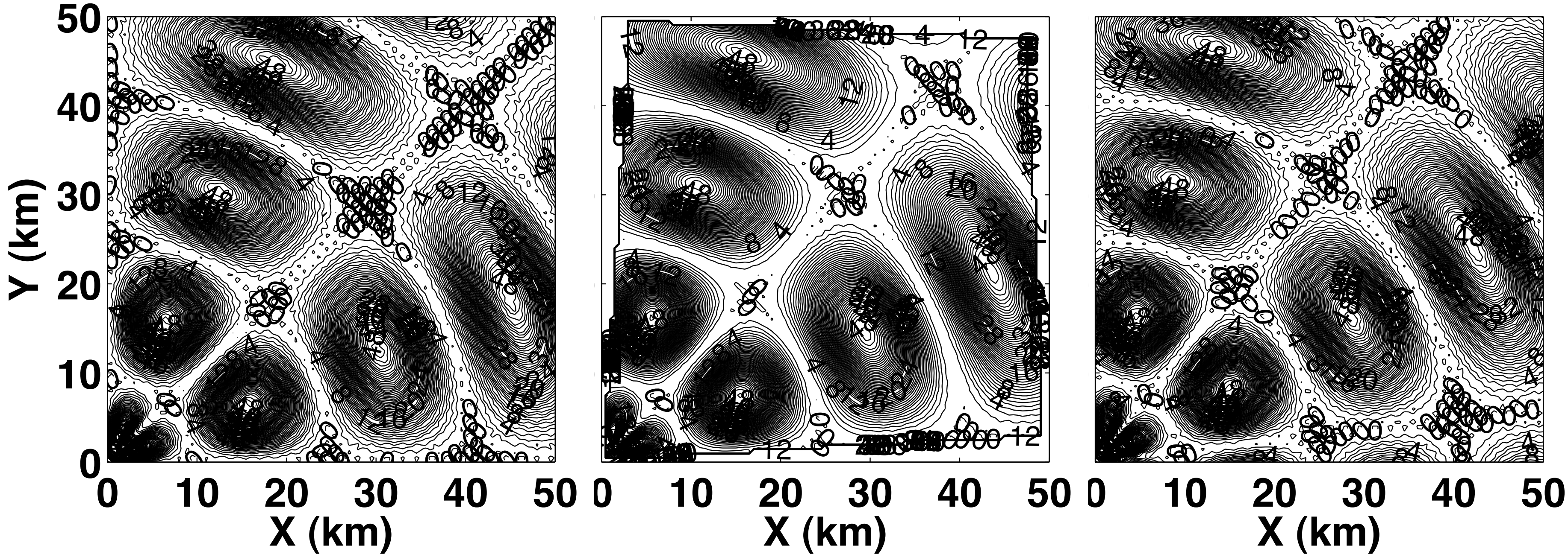
Step 3: Interpolate Z data to the end-points of each trajectory.

Step 4: Evaluate analytical solution for Z.

Advection of reflectivity blobs in a solid body vortex

Z at first input time ($t = 0$ min)

Z at second input time ($t = 6$ min)

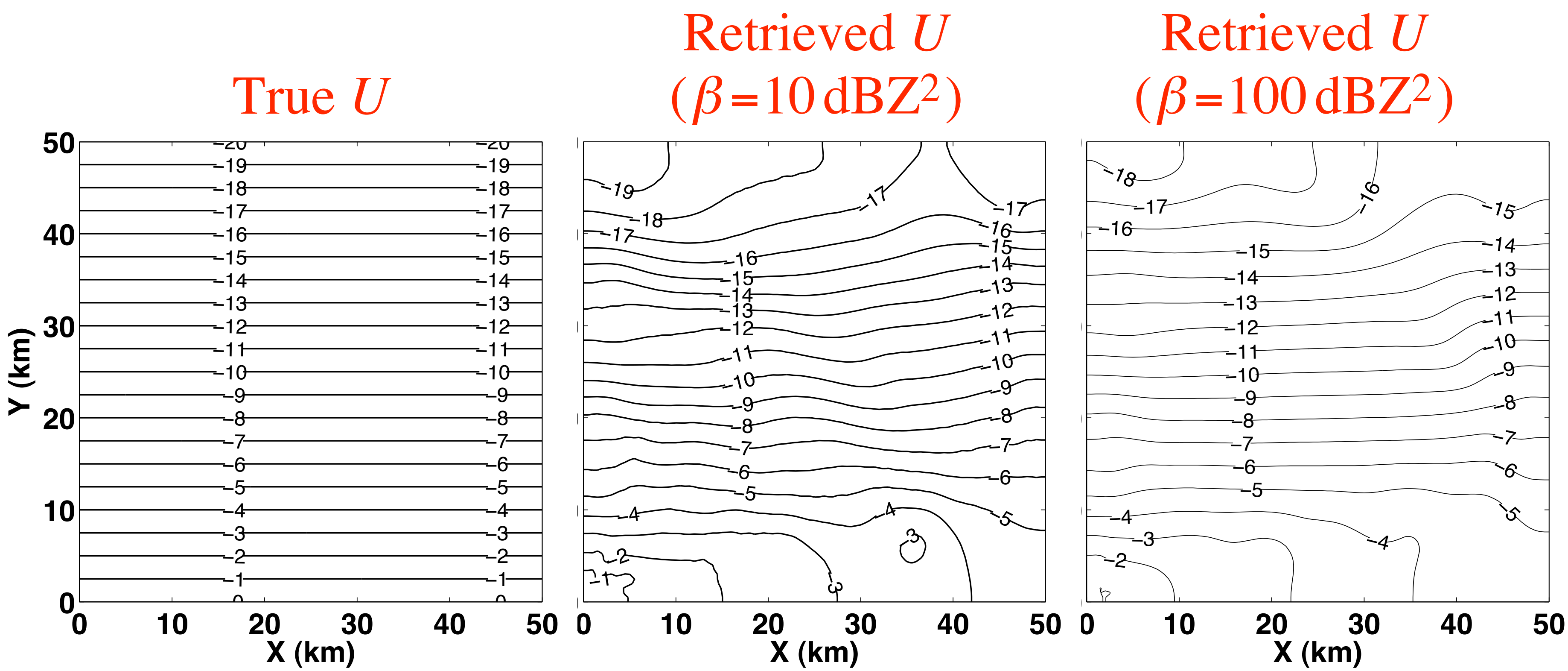


⇑

Advection-corrected Z at middle time ($t = 3$ min)

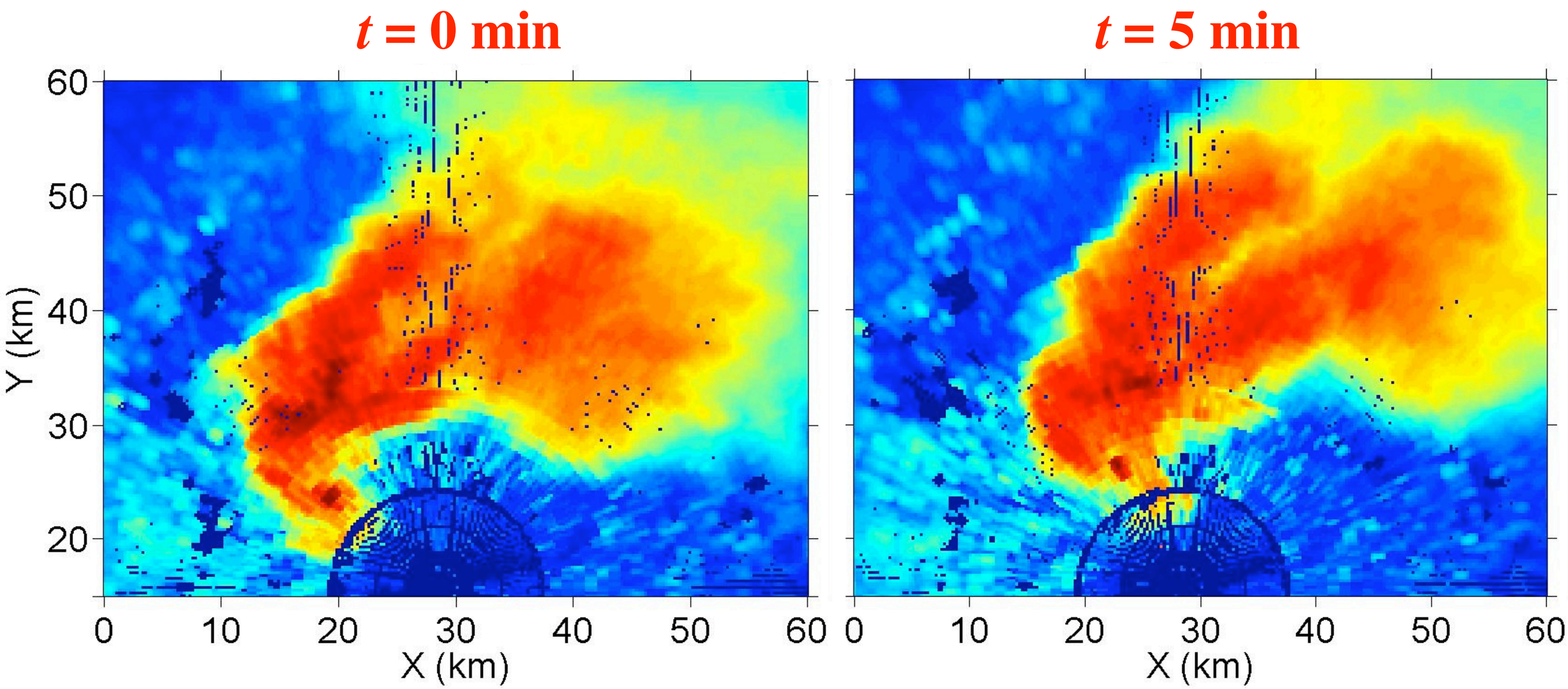
Advection-corrected Z is from a $\beta = 100\,\mathrm{dBZ}^2$ experiment.

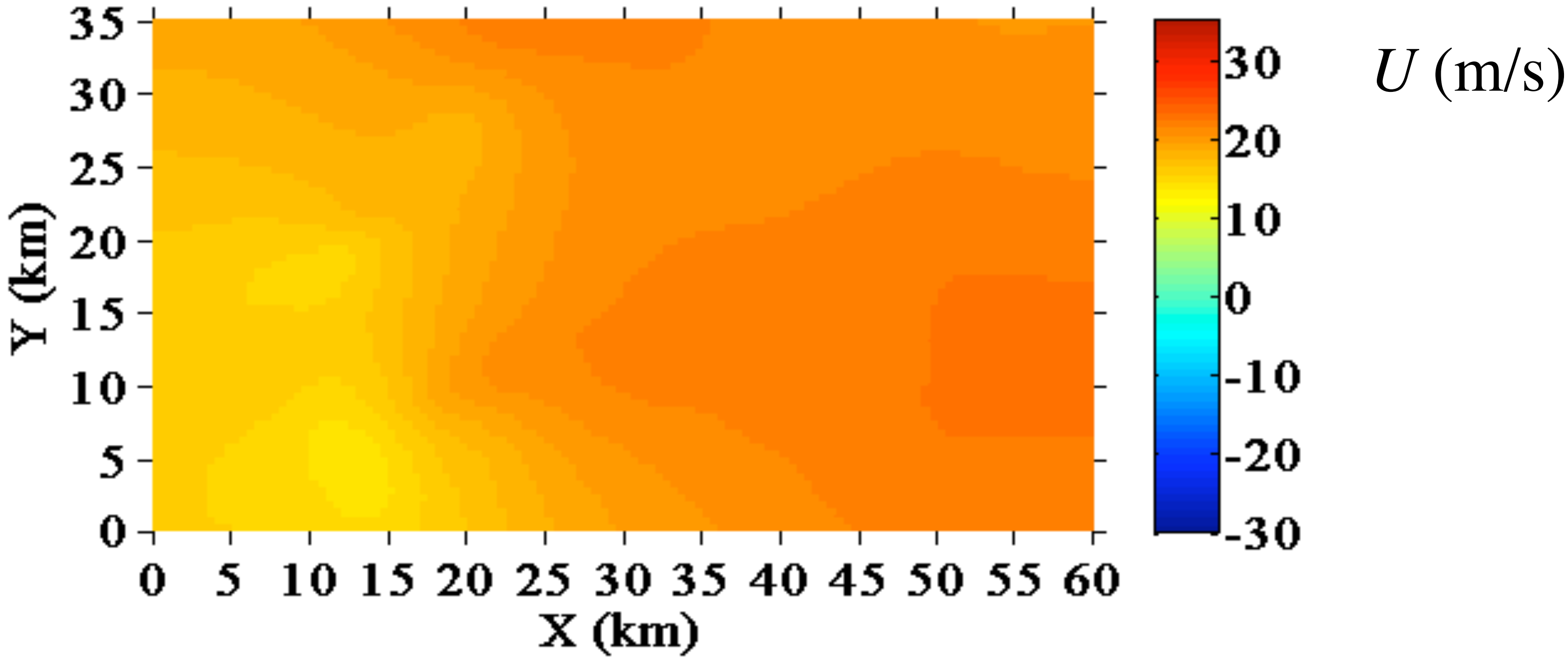
Retrieved *U* in solid body vortex experiment

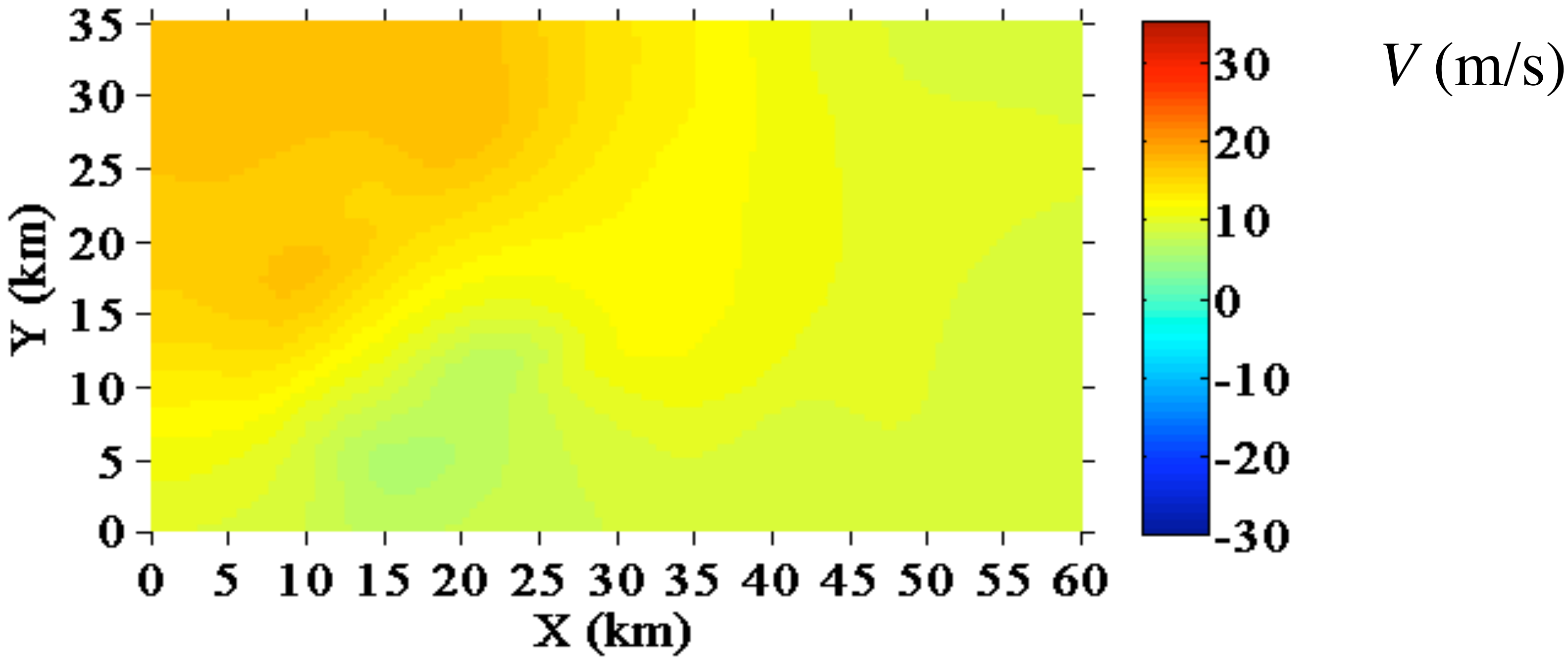


Test case: Oklahoma supercell storm, 8 May 2003

Input data: Two scans of WSR-88D radar reflectivity (KTLX radar)

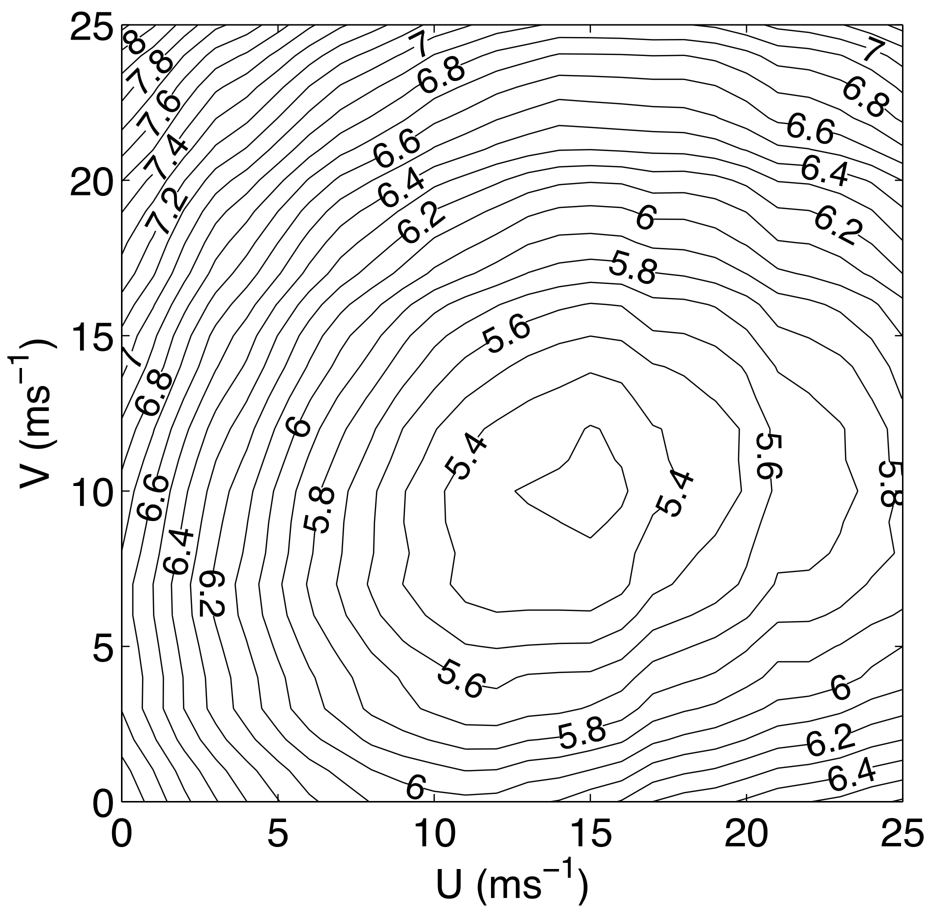



35
30
25
20
15
10
5
0
Y (km)
0 5 10 15 20 25 30 35 40 45 50 55 60
X (km)
30
20
10
0
-10
-20
-30
U (m/s)


35
30
25
20
15
10
5
0
Y (km)
0 5 10 15 20 25 30 35 40 45 50 55 60
X (km)
30
20
10
0
-10
-20
-30
V (m/s)

Tests using 8 May 2003 TDWR data

Results with TDWR data were similar to those with WSR-88D data, but since TDWR data were available every ~1 min, could compare retrieved *Z* with true *Z*. **RMS error in *Z* (~ 4.5 dBZ) was less than RMS errors in Z obtained in any constant *U*, *V* experiment:**



Use of the anelastic vertical vorticity equation in dual-Doppler wind analysis

Taking $\hat{k} \cdot (\nabla \times$ anelastic equations of motion) yields an equation for the evolution of the vertical vorticity ($\zeta = \partial v/\partial x - \partial u/\partial y$):

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\zeta = \frac{\partial u}{\partial z}\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\frac{\partial w}{\partial x} - \zeta\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \qquad (15)$$

No baroclinic term in here (no p or ρ). Baroclinicity is very important in convective storms, but the baroclinic vector is mostly horizontal.

Since (15) relates *w* to *u* and *v*, it can be used as a constraint in dual-Doppler wind analysis (Protat & Zawadzki 2000; Protat et al. 2001; Mewes & Shapiro 2002; Liu et al. 2005; Shapiro et al. 2009).

Contending with unsteady term in vorticity equation

Method 0. Ignore the term

$$\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right) = 0$$

Method 1. Impose frozen turbulence constraint

Impose frozen-turbulence constraint (say, with spatially variable U, V):

$$\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right) = -U\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right) - V\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right). \qquad (16)$$

Method 2. Impose frozen turbulence with intrinsic evolution

As air parcels translate, let their vorticity change linearly with time. Requires estimates of the vorticity fields at two time levels – get these fields from a simplified dual-Doppler analysis without w.

3DVAR analysis with vorticity equation constraint

Seek u, v, w that minimize the sum of errors in the analysis constraints:

$$J \equiv \iiiint \left(\alpha_1 O_1^2 + \alpha_2 O_2^2 \right) dr\, d\theta\, d\phi\, dt \quad + \iiint \left(\delta\, \varepsilon_m^2 + \gamma\, \varepsilon_v^2 + \beta_1 S_1 + \beta_2 S_2 + \beta_3 S_3 + \beta_4 S_4 \right) dx\, dy\, dz. \tag{17}$$

O_1, O_2: Differences between analyzed and observed v_r data.

ε_m: Residual in mass conservation equation.

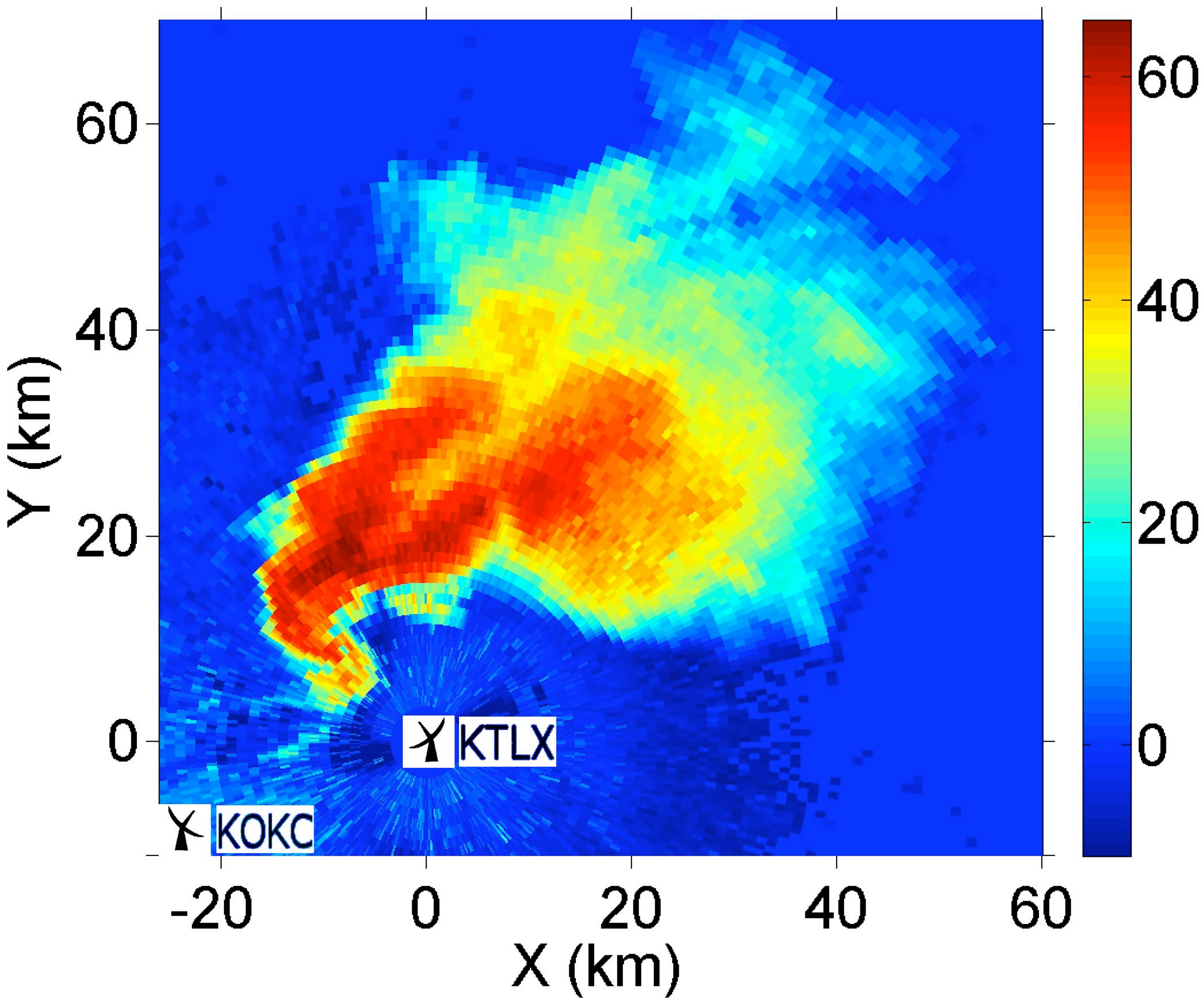
ε_v: Residual in anelastic vertical vorticity equation.

$S_1 - S_4$: Squared spatial derivatives of u, v, w (smoothness terms).

$\beta_1 - \beta_4$: Smoothness weights.

J is minimized with a conjugate-gradient algorithm.

Test case: Oklahoma supercell storm, 8 May 2003



Data denial experiments

Control Run ("truth")

No vorticity equation constraint but all other constraints turned on. Radial wind data used throughout the analysis domain (as far down to the ground as possible; generally down to 100 - 200 m AGL)

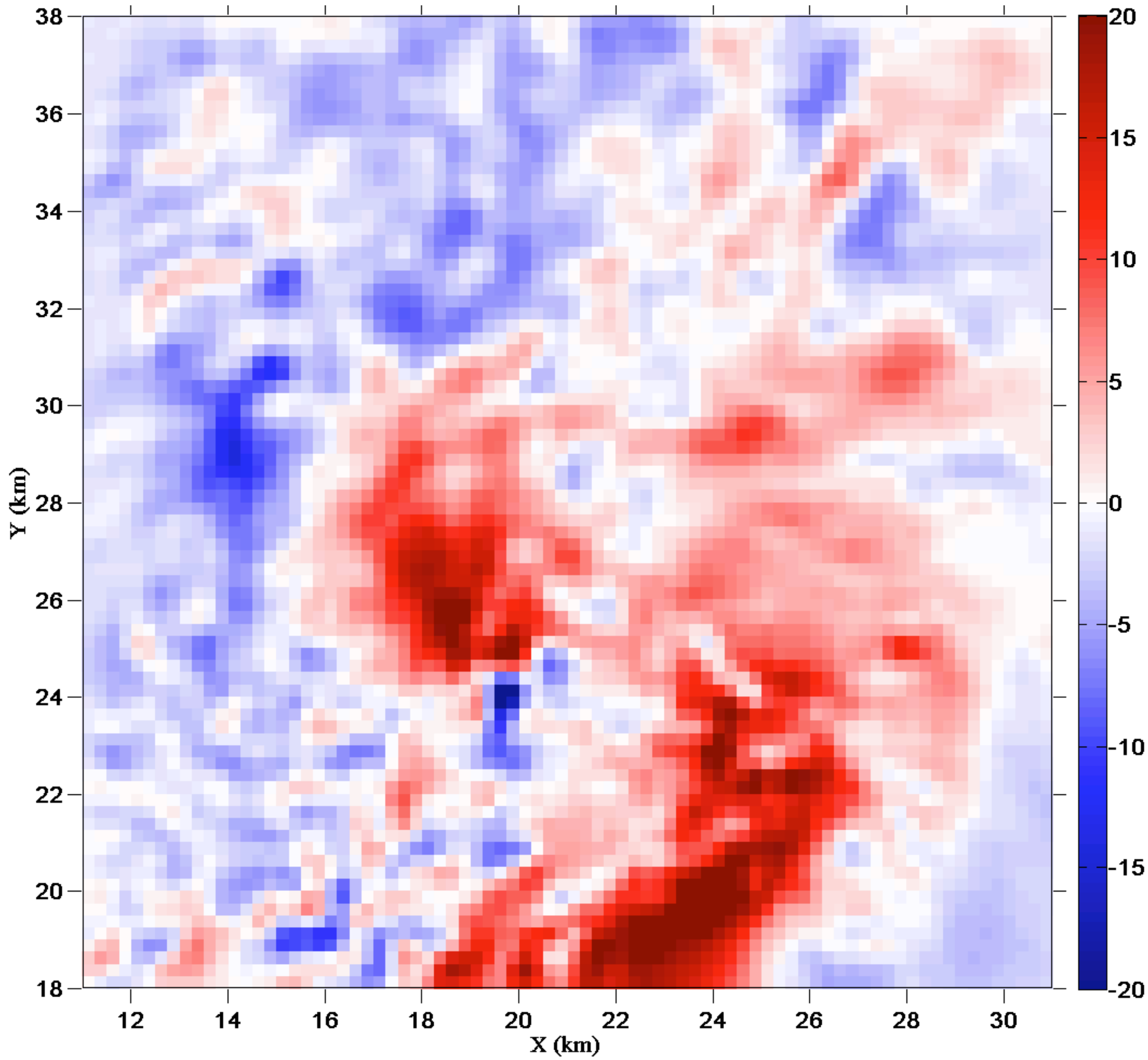
Data Denial Experiment 1: NOVORT

Radial winds thrown out for $z < 1$ km. Otherwise, experiment is same as control run (no vorticity equation constraint).

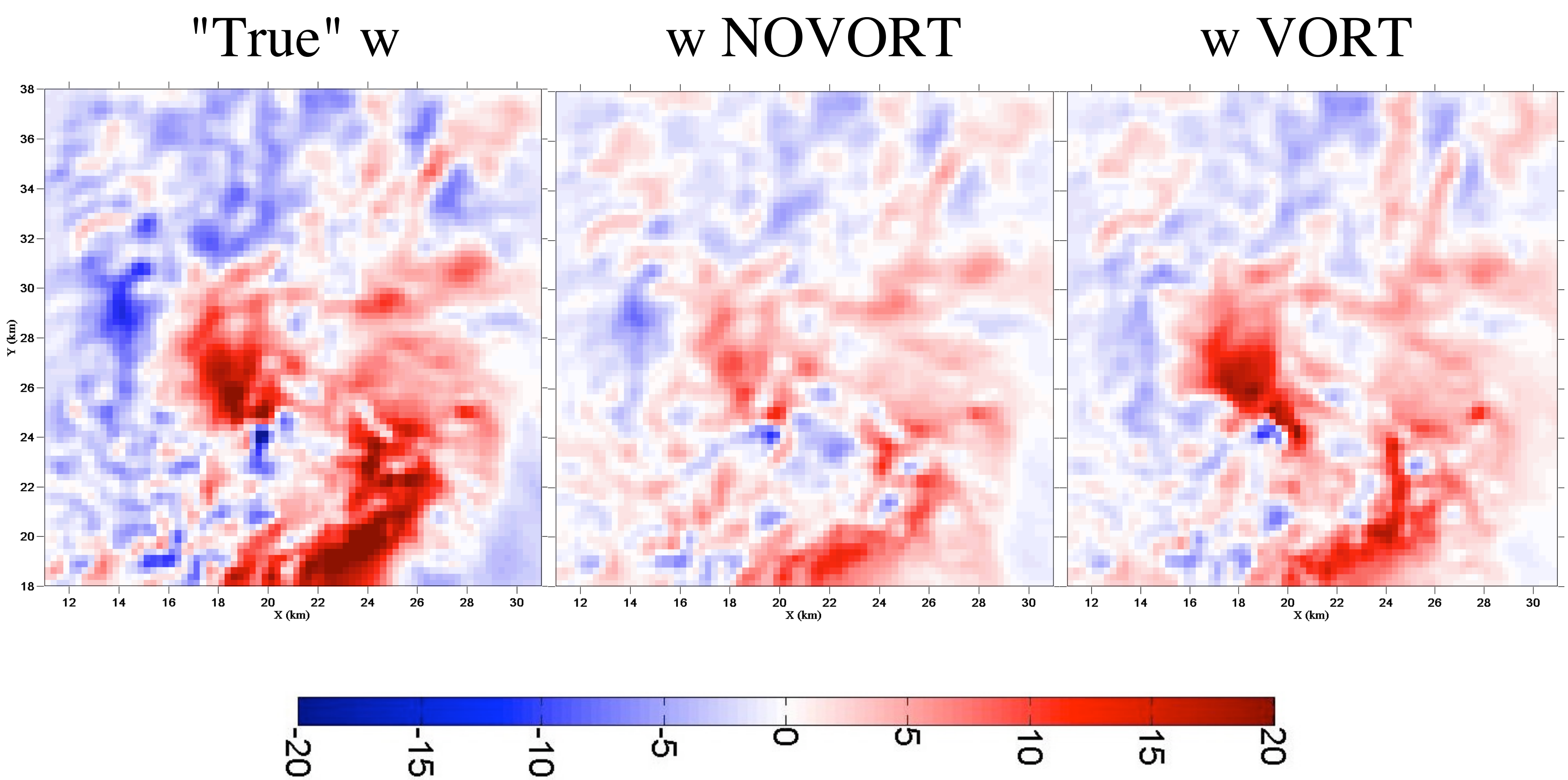
Data Denial Experiment 2: VORT

Radial winds thrown out for $z < 1$ km. The vorticity equation constraint is turned on but with no provision for evolution.

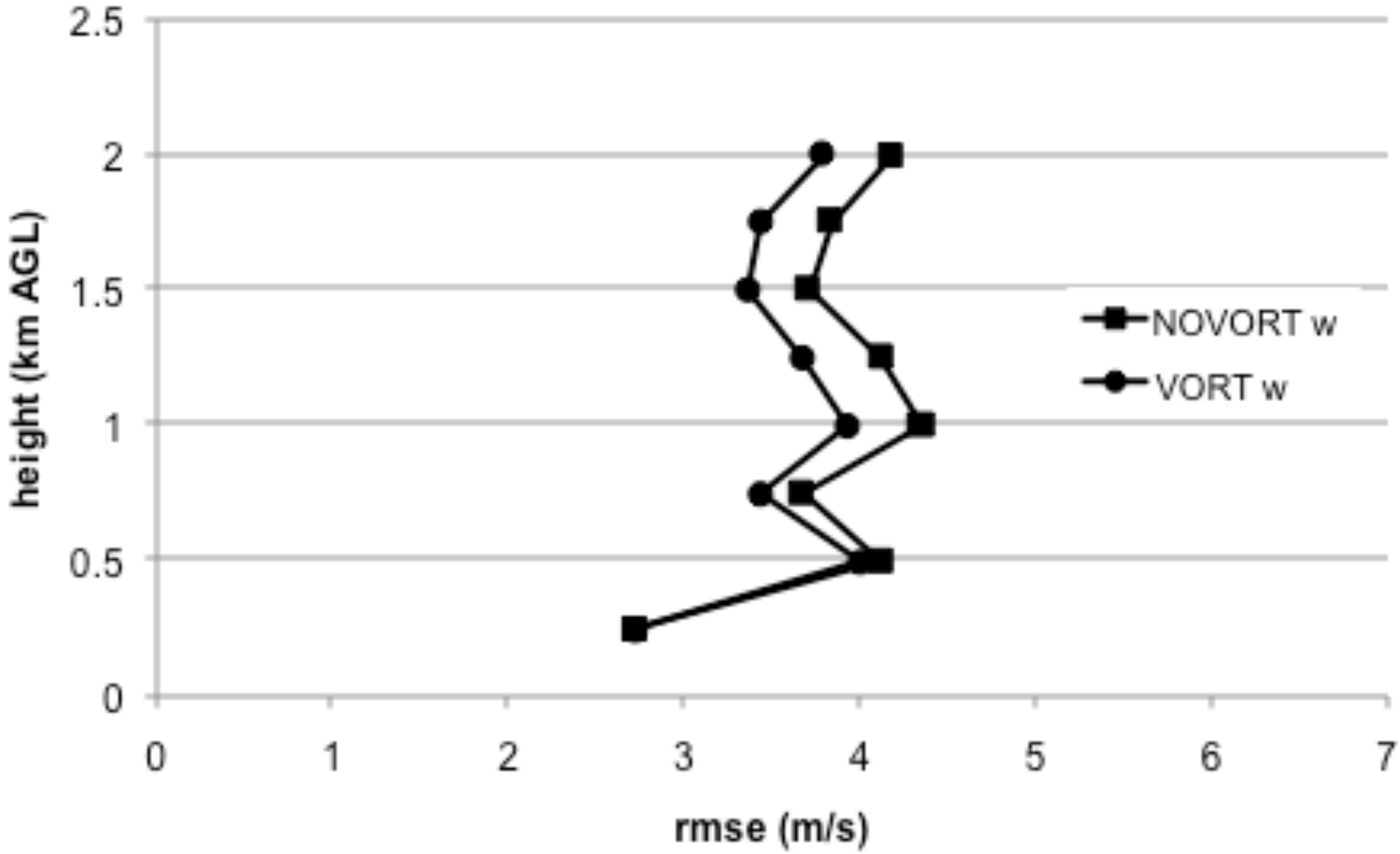
"True" *w* (m/s) at *z* = 1.75 km AGL



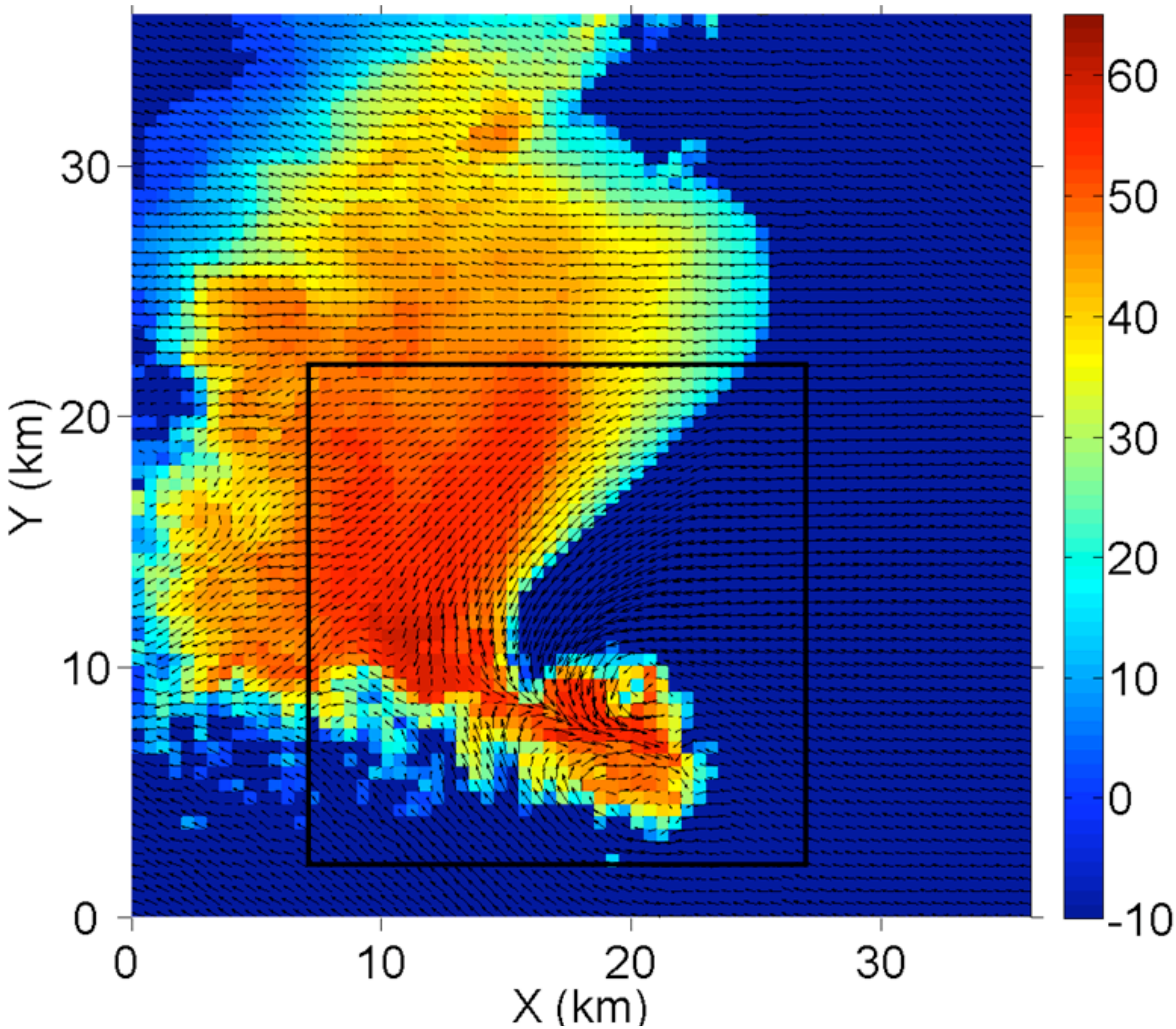
Impact of vorticity constraint: *w* (m/s) at *z* = 1.75 km AGL

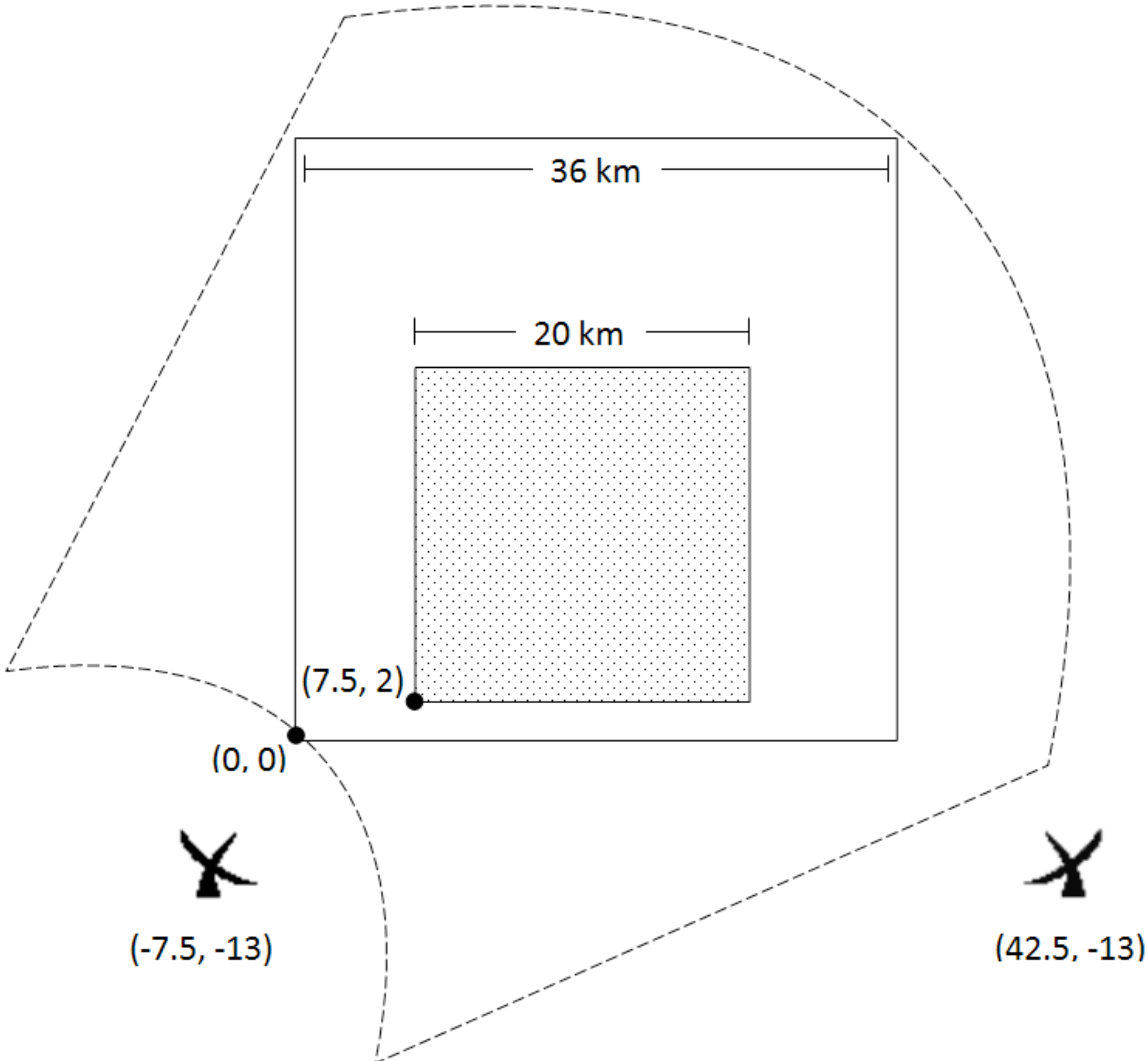


RMS error in *w* versus height



Tests using Advanced Regional Prediction System (ARPS) supercell storm data





36 km
20 km
(7.5, 2)
(0, 0)
(-7.5, -13)
(42.5, -13)

Data denial experiments

Control Run ("truth")

w field output from ARPS run

Data Denial Experiment 1: NOVORT

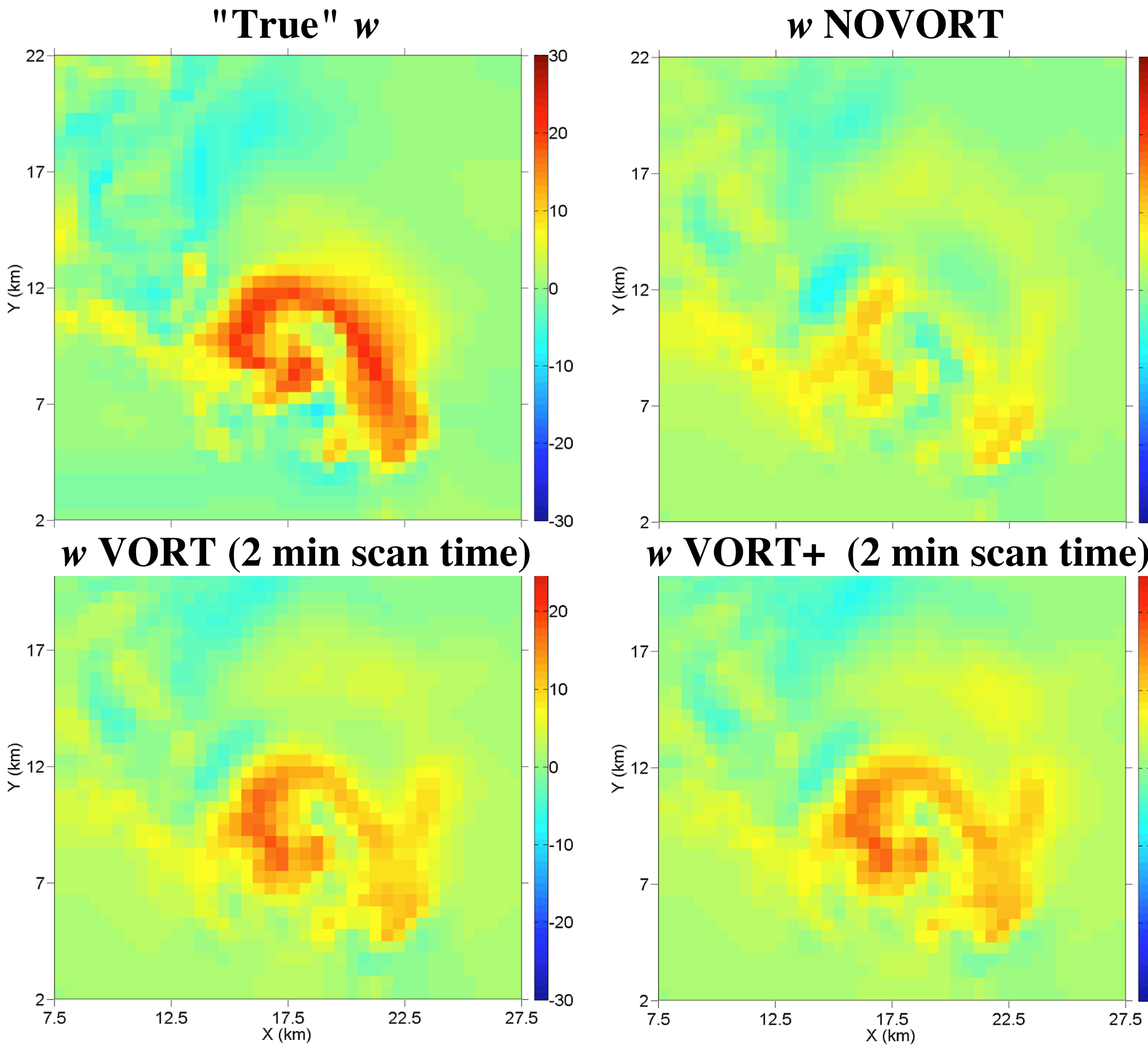
Radial winds thrown out for z<1.5 km. Data, mass conservation and smoothness constraints imposed. No vorticity equation constraint.

Data Denial Experiment 2: VORT

Radial winds thrown out for z < 1.5 km. As in NOVORT but now the vorticity equation constraint is imposed. Spatially variable frozen turbulence applied in unsteady term but with no evolution.

Data Denial Experiment 3: VORT+

Radial winds thrown out for z < 1.5 km. As in VORT but now evolution is accounted for (crudely) in unsteady term.


"True" w
w NOVORT
w VORT (2 min scan time)
w VORT+ (2 min scan time)
Y (km)
X (km)
7.5
12.5
17.5
22.5
27.5
2
7
12
17
22
30
20
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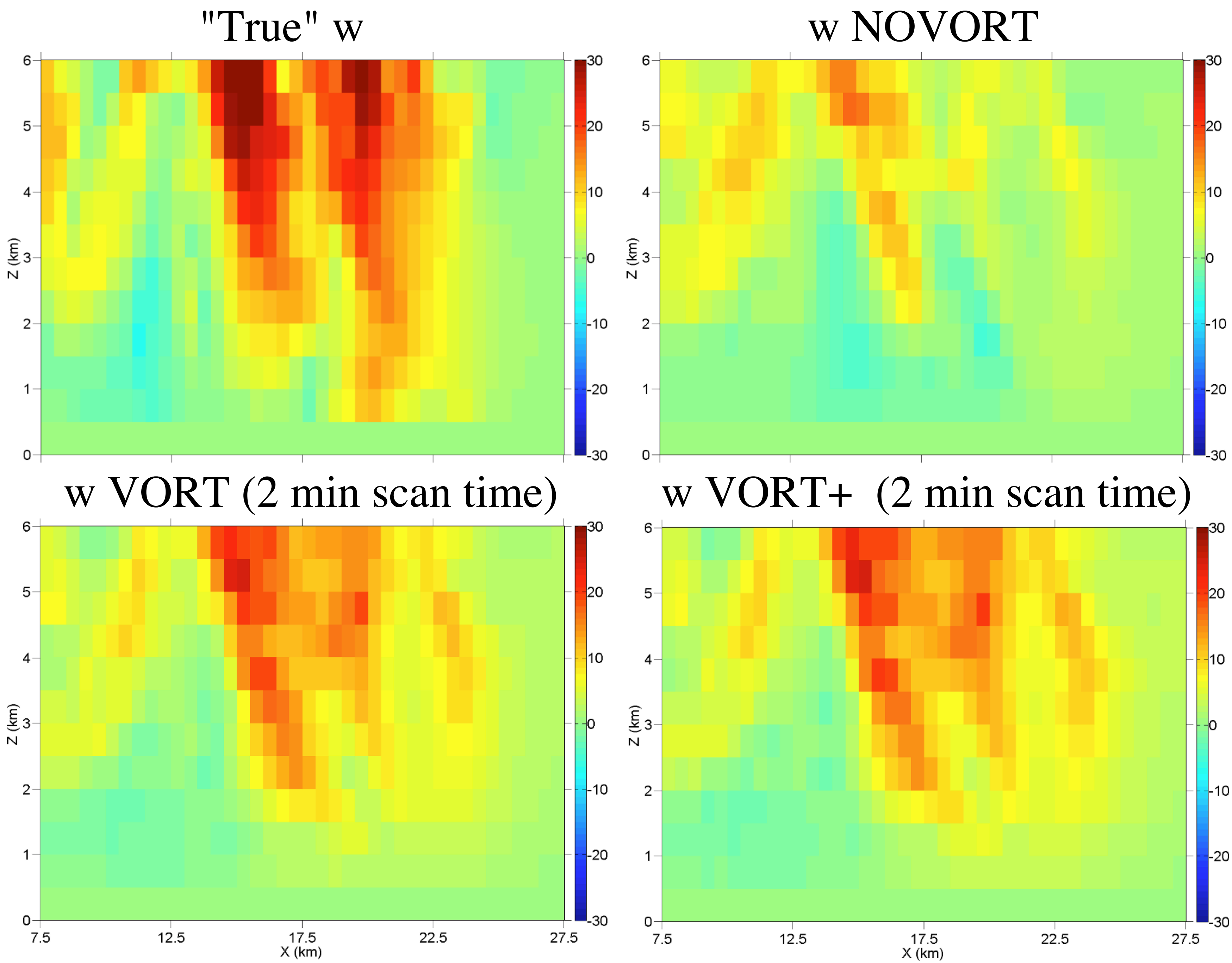
0
-10
-20
-30


"True" w
w NOVORT
w VORT (2 min scan time)
w VORT+ (2 min scan time)
Z (km)
X (km)
7.5
12.5
17.5
22.5
27.5
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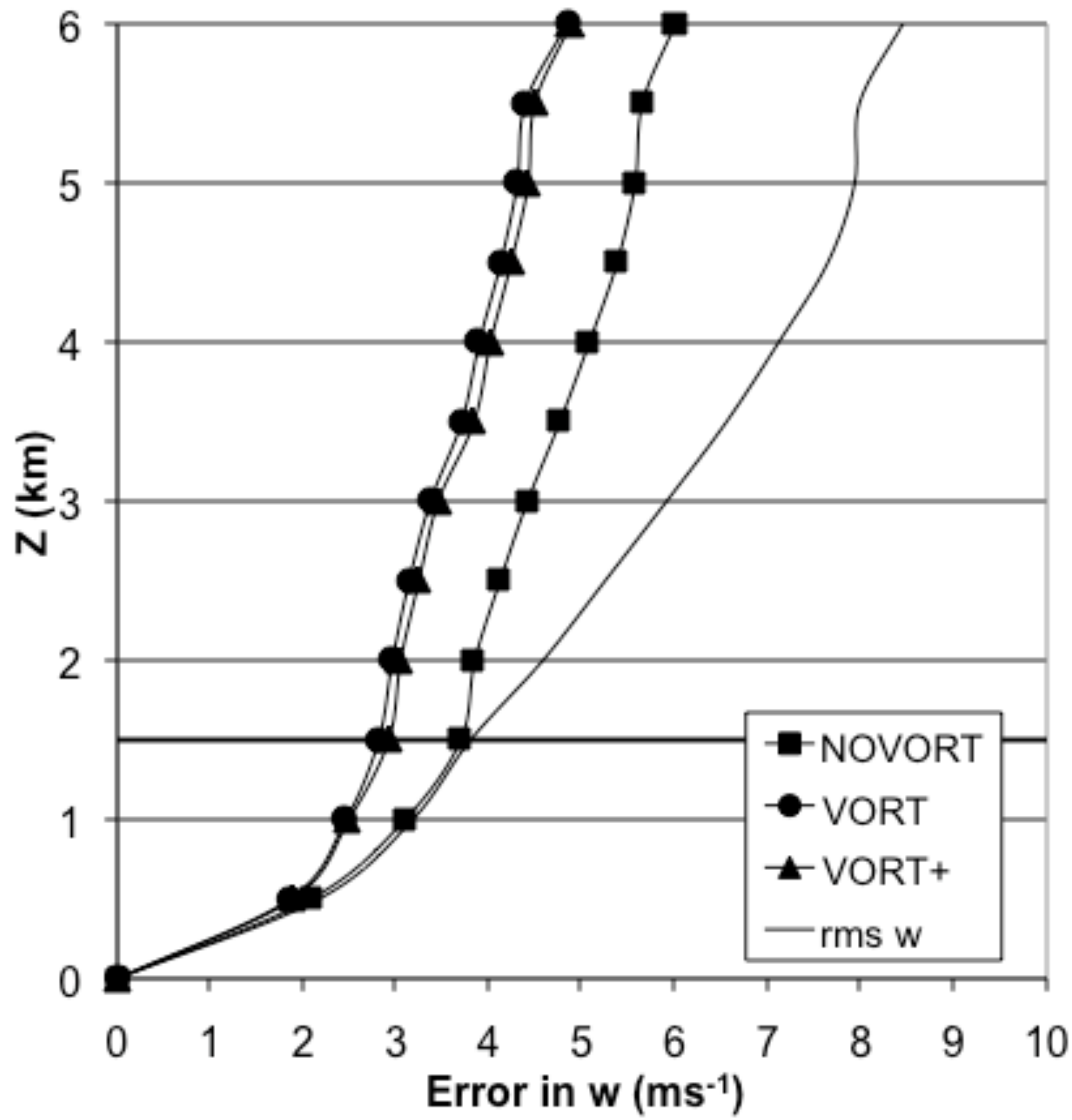
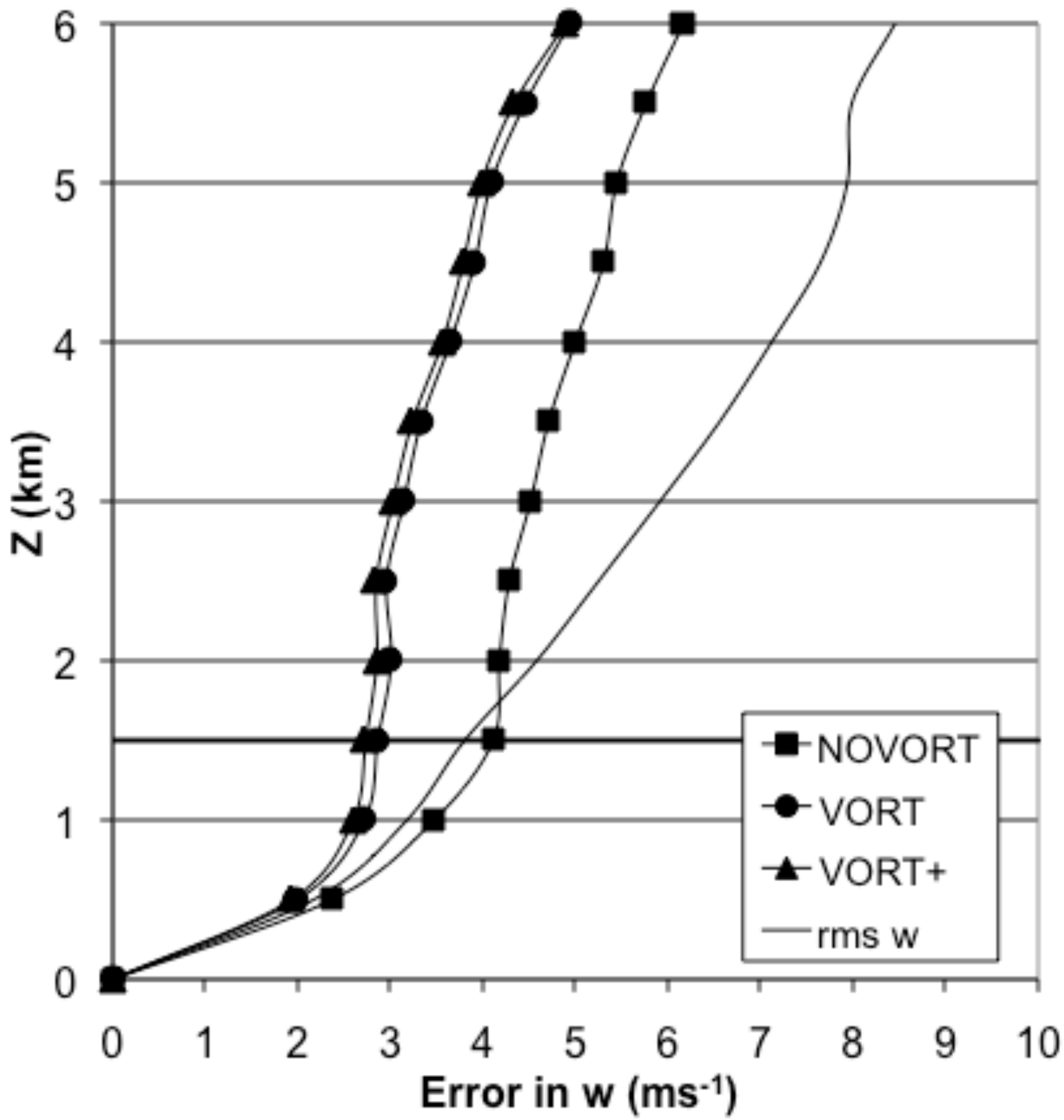
1
2
3
4
5
6
30
20
10
0
-10
-20
-30

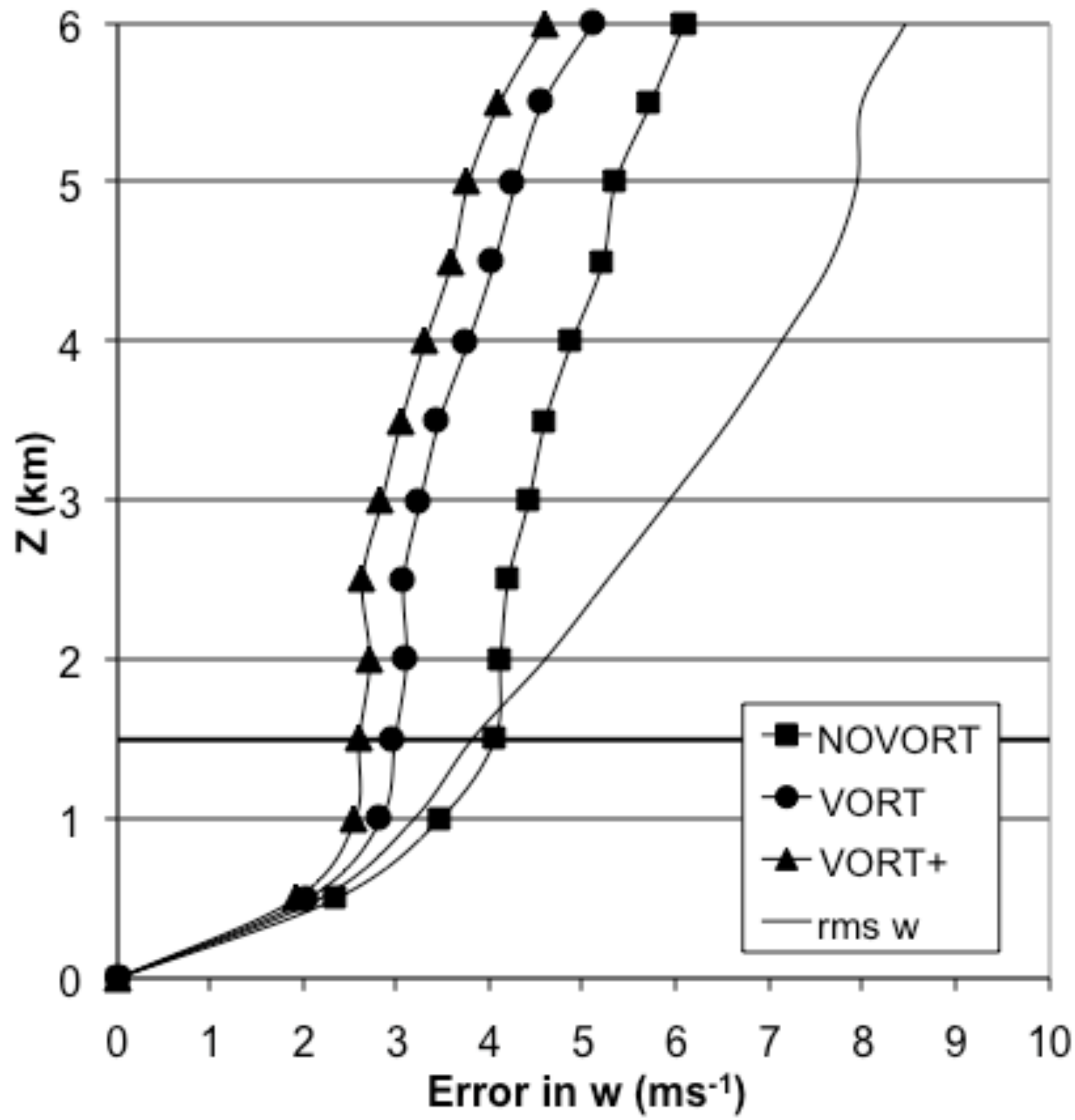
5 min volume scans



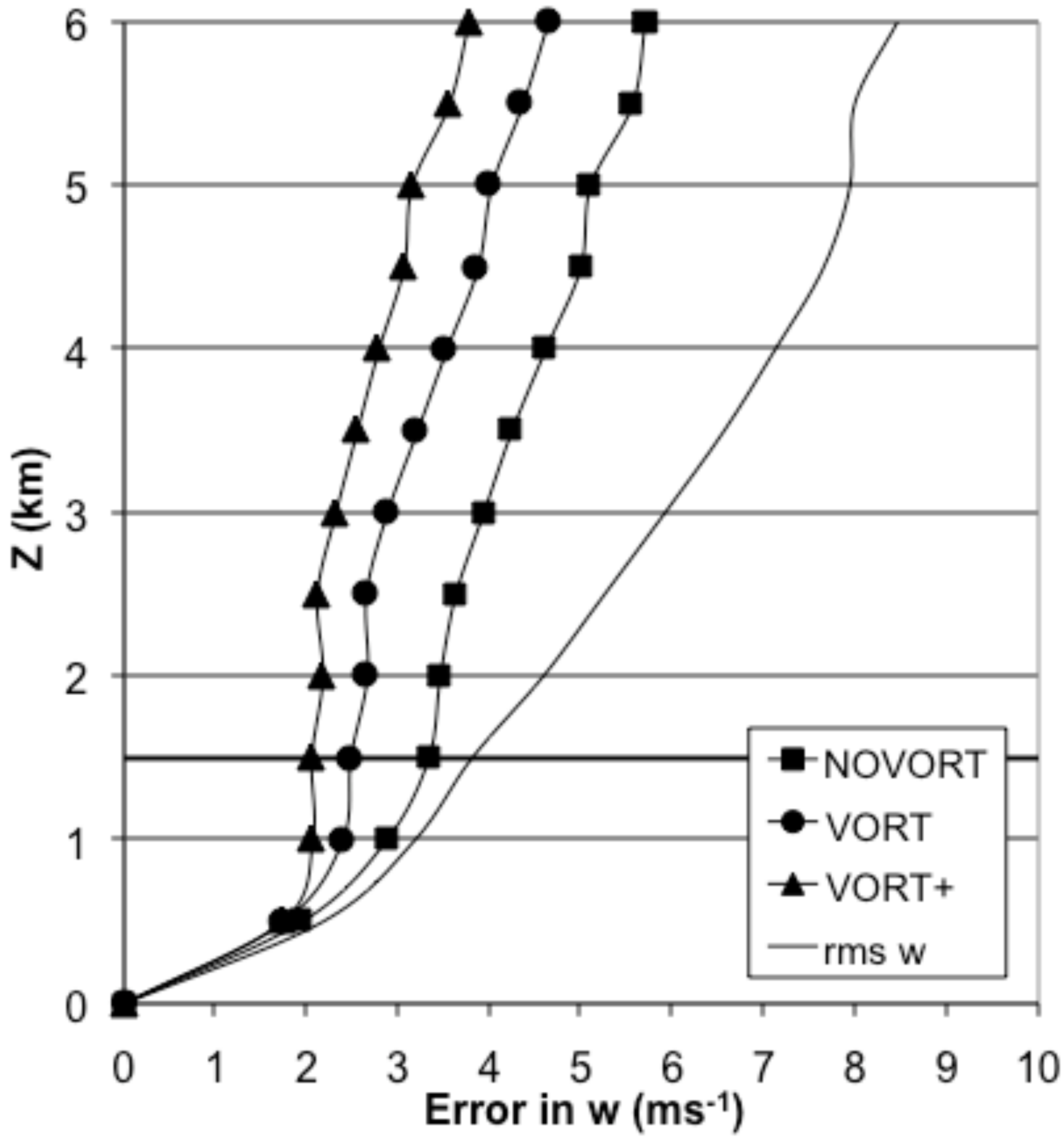
2 min volume scans



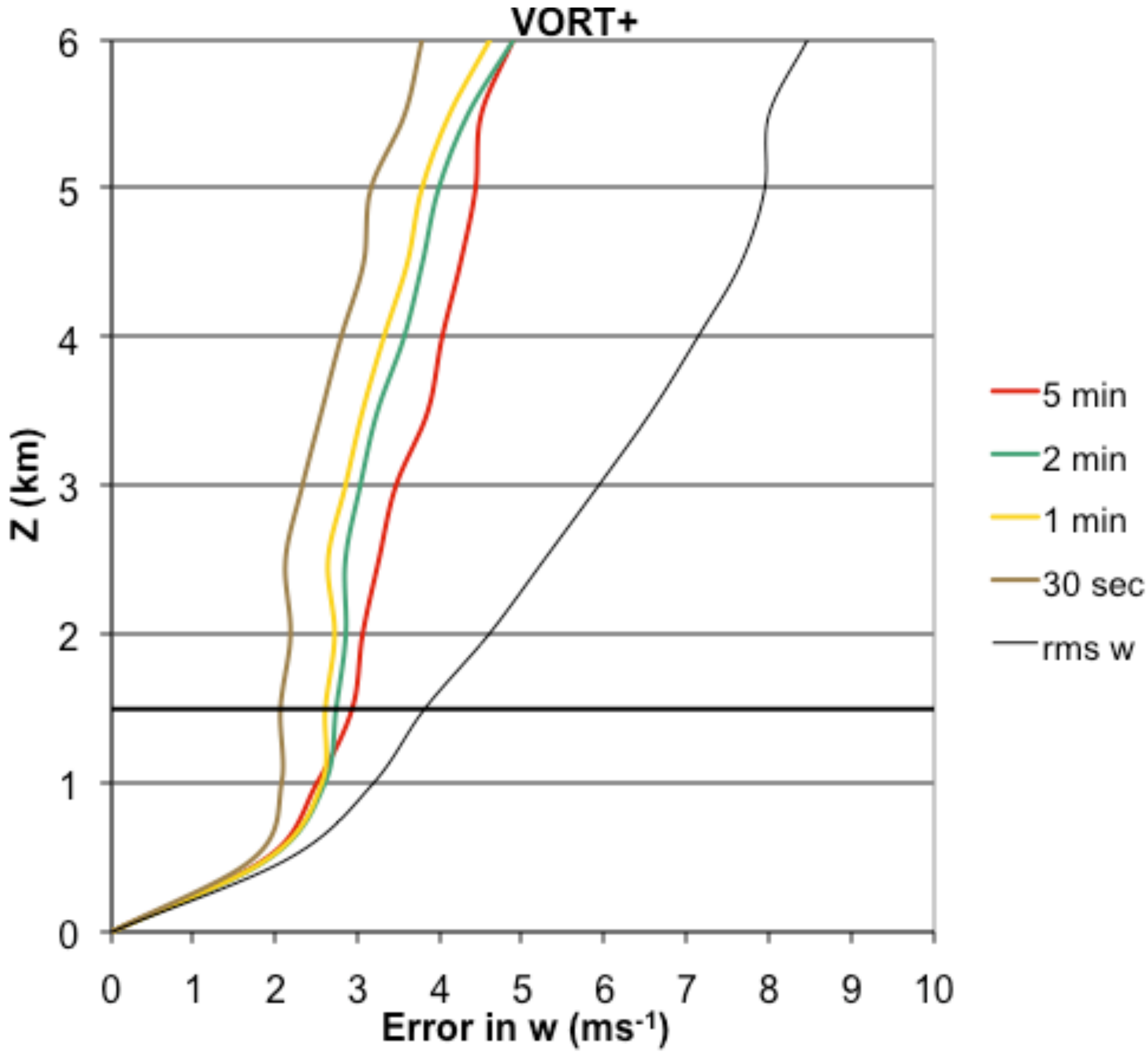
1 min volume scans



30 sec volume scans



Focus on VORT+ results



Future work

1. Advection Correction

Derive a spatially variable advection-correction procedure based on radial wind data, i.e., based on Euler-Lagrange equations arising from minimization of $D^2(rv_r)/Dt^2 = 0$ subject to smoothness constraints.

2. Dual-Doppler wind analysis

Improve estimates of vorticity tendency by using improved spatially variable U, V fields (see above).

Improve estimates of vorticity tendency by using rapid scan radar data (volume scans ~ 1 min or less), e.g. from PAR radar, CASA radars, SMART-R radars, DOW radars.